“CAN’T GET NO SATISFACTION”.

Firstly, let me define what I mean by “**satisfaction**”. I do not want to imply any connection with the word “**satisfactory**” which generally means “**just OK**”.
I want to define “**satisfaction**” as something that gives us “**immense pleasure**”.

After correctly working out a trigonometric equation involving considerable manipulation, a fairly average year 13 girl recently said out loud **“I feel so smart now!”** She obviously experienced that wonderful feeling of **self satisfaction, fun, enjoyment and immense pleasure** from doing this task **by herself**.

There seems to be a growing feeling that questions involving “just skills” are no longer important, even amongst some relatively influential teachers!
I strongly believe that mathematical skills or logical procedures are **absolutely essential to all progress and understanding.** However one point, that these people are completely missing, is that students **enjoy** mastering these essential skills, techniques, logical processes or whatever you want to call them.

Recently, I had a whole class of Year 12 students simply learning to master the factorisation of trinomials of the form $3x^2 - 7x - 6$. When they had mastered the logical process, the feeling of **self satisfaction, achievement and enjoyment** from every, single student was very apparent from the looks on their faces!

Why are some people obsessed with “**real life**” problems? Mathematics is not **just** about modelling and solving so called “real life” problems. If all Mathematics had evolved from just “real life” problems we would only have a small fraction of the knowledge we have today.
I rather like this adulterated version of another famous saying:
“Applied Mathematics is a Problem looking for a Solution
but Pure Mathematics is a Solution looking for a Problem”.

Mathematicians study things because they want to find out **all there is to know** about those things!
Let me give a simple example: Suppose a young mathematician has only been familiar with linear equations and so expects problems to have just one solution but then he/she meets a new type of problem like this:

“If I square a number then add 12, the answer is 8 times the original number. Find the number.” (ie $x^2 + 12 = 8x$)

he/she would be very surprised that there are 2 different solutions to this problem. (The answer could be 2 or 6 of course.)
The young mathematician would start investigating **(or be lead by a good teacher)** and find out that the basic key to these equations lies in this logic:

$$\textit{If} \quad a \times b = 0 \textit{ then } \quad a = 0 \textit{ or } b = 0$$

$$\textit{leading to : if } \quad (x - 3)(x + 5) = 0 \textit{ then } \quad x = 3 \textit{ or } x = -5$$

which in turn leads to $x^2 - 8x + 12 = 0$ (and hence the **need** for **factorising!**)

When the young mathematician is ready, equations such as: $(x - 1)^2 = 3$ can be studied leading to "*completing the square*" method and, assuming he/she has good algebra skills the quadratic formula can be proved by "*completing the square*" on $ax^2 + bx + c = 0$
This of course leads to other investigations because these new equations seem to have **different types** of solutions.

He/she would find that one type, like $x^2 - 8x + 12 = 0$ has 2 rational solutions then a slight change such as $x^2 - 8x + 16 = 0$ means the equation would only have 1 rational solution.
Another slight change produces an equation like $x^2 - 8x + 14 = 0$ which would have 2 irrational solutions and some equations such as $x^2 - 8x + 17 = 0$ seem to have no real solutions at all. (This would later lead the young mathematician off into the realm of complex numbers.)

The next step would be to find out **WHY**.
It is actually a huge mental leap forward for many young students to find out that the solutions of f(x) = 0 are where the graph of y = f(x) crosses the x axis!

Lots of mathematics is done ***for its own sake*** simply because it is **interesting** and **enjoyable**. The young mathematician wants to know **all about this new type of equation** and all the different cases. At the time, the young mathematician is not concerned about the **applications** of these equations to some real life problem.
Of course, subsequently it will be found there are many applications!
This is where the mathematician would get this feeling of ***great self satisfaction*** again.

Algebraic manipulation is an **intellectual pursuit**. When you can do it, you get this feeling of ***great satisfaction*** that I keep referring to.

The trigonometric manipulation involved in proving identities is in the same intellectual league as playing chess at a high level.

Before you can become good or even reasonably competent at golf, skiing, driving, rugby, soccer, netball, tennis, writing, languages or maths, **you need the basic skills**.
Where is the **joy** and **self-satisfaction** in solving $\sin(2x) - \cos(2x) = \frac{\sqrt{3}}{\sqrt{2}}$

by just entering it into a graphics calculator?

Students (at least at my school) want to be challenged and they also want the satisfaction which comes from overcoming those challenges.

Using a graphics calculator to solve a quadratic equation simply involves entering ***a, b and c*** and pressing "**solve**". **Where is the joy in this?** How does this promote the spirit of wonder or achievement? **I think it actually stifles it for most students!**

Someone with considerable influence in mathematical circles (excuse the pun) said recently, "We don't multiply 345 by 62 by long multiplication any more. We just use a calculator. We should learn to use new technologies as they become available." This is true but the real point is that **we must be still able** to multiply 345 by 62 if we needed to, whereas we are rapidly getting to the stage when the **only way** people will be able to solve equations will be on a calculator because the necessary algebraic skills and understanding are being lost or considerably neglected and under-rated.

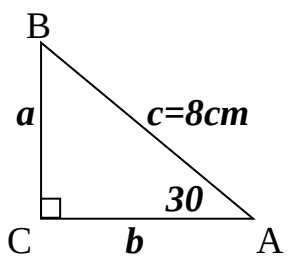
To put it simply, this is an argument about doing things the "THINKING WAY" verses the "NON-THINKING WAY".

There is no problem in using calculators for numerical calculations, finding $\tan(49^0)$, finding $\log_{10}(1.82)$, finding $e^{1.67}$ etc. The particular use which is **crippling** students' expertise and understanding is using the calculators **"to solve equations"**.

For some people the basic techniques of calculus are becoming irrelevant!
Students can be taught to find the gradient of $y = x^2$ ***at*** $x = 3$ ***without knowing how to differentiate*** x^2 ***and to find definite integrals such as:*** $\int_2^4 3x^2\,dx$
without even knowing how to integrate $3x^2$.

Some students are actually being taught to find the equation of a line through two given points using the ***linear regression function*** on their graphics calculators which means they don't need to know about ***gradient and y intercept*** at all!

It would be easy to make a "calculator trigonometric function" as follows:



Simply enter all values you are given from the diagram

a	*b*	*c*	∠*A*	∠*B*	∠*C*
		8	30		90

then we simply press **"solve"** and **ALL** unknowns in the triangle are calculated:

a	*b*	*c*	∠*A*	∠*B*	∠*C*
4	*6.9*	8	30	*60*	90

Using this idea, who would need to bother knowing about **sine, cosine and tangent**? Would we be saying that a person who uses the above "program" can **do** trigonometry?
Is this just another example of the direction mathematics education is heading?

When the USE of a calculator REPLACES UNDERSTANDING then it is really ABUSE of the calculator.

Consider the following type of question commonly asked at level 1algebra:

The cost of 2 adult tickets and 3 children's tickets to a concert is $700.
The cost of 1 adult ticket and 2 children's tickets is $400.
Find the cost of an adult ticket and the cost of a child's ticket by solving the simultaneous equations: $2a + 3c = 700$
$a + 2c = 400$

I find the above type of question to be **insulting** to our students.

It pretends to be a "**problem**" but actually provides students with the necessary equations which the student could have obtained from the information given, making that information irrelevant!

The student then "enters" ***a, b*** and ***c*** for both these equations into their graphics calculators which tells them ***a = 200 and c = 100.***

Students really **DO NOT** feel any **SATISFACTION** by doing this sort of thing.

What mathematics has been done?
What understanding has been shown?
What value does this have?
Who on earth are we trying to fool or impress with problems like this?

I heard one teacher who is teaching with the C.A.S. calculators say, "My students just do the algebra mentally. They find the calculator just holds them back!"
This reminds me of a joke I saw perhaps 40 years ago.
It involved a primary school teacher saying to an inspector "Jane can do additions and subtractions perfectly in her head but she just can't do them using the little rods yet!"

I have heard C.A.S. jokingly referred to as standing for "**C**rippling **A**lgebra **S**kills". Unfortunately, I fear this is not far from the truth.

Proper **use** of these calculators should **aid understanding** but **abuse** of these calculators will **replace understanding.**
Similarly, proper **use** should **ENHANCE algebraic expertise** but **abuse** totally **neglects** and **under-rates algebraic expertise**.

Philip Lloyd