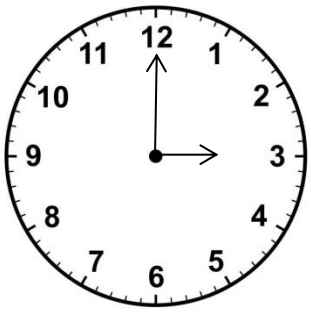
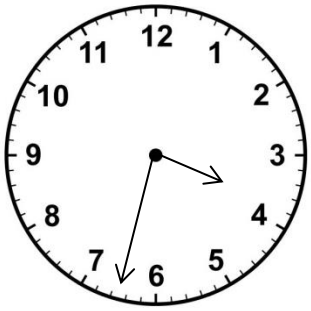
Special Angles between the Hour Hand and the Minute Hand of a clock.

(Angles will be measured in the clockwise direction from the 12 position.)



Obviously, the angle here is 90 degrees.



The type of question I was thinking about is "***When will the next time that the angle will be 90 degrees***?"

It is definitely not at 3:30 because the hour hand will have moved as seen here.

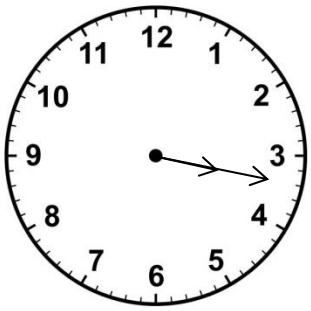
To calculate the times necessary for such questions, we need the work out the rate of change of the angle that each hand is rotating.

The minute hand rotates 360 degrees in 60 minutes so this means it rotates precisely 6 degrees per minute.
In "t" minutes, the minute hand rotates 6t degrees.

The hour hand rotates 360 degrees in 12 hours or 720 minutes, so this means that the hour hand rotates ½ a degree per minute.
(This is also 30 degrees per hour.)
In "t" minutes, the hour hand rotates ½ t degrees.

In the above example at 3 pm we can say that M, the minute hand's angle is M = 0 degrees and H, the hour hand is H = 90 degrees.

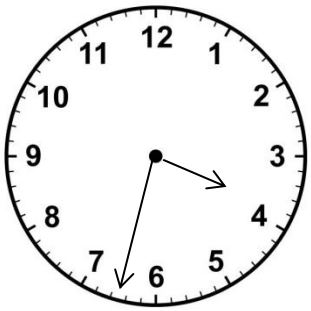
At t minutes later, the minute hand is at M = 6t degrees and the hour hand is at H = 90 + ½ t because it starts off at 3pm



"When will the hands be in line?"

The minute hand will **in line** with the hour hand when the angles are equal!

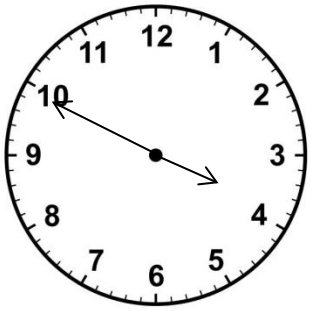
Putting $6t = 90 + \frac{1}{2}t$
we get $5.5t = 90$
so $t = 16.3636\ldots$min
$= 16$ min 21.8 sec approx. (i.e. past 3 pm)



"When will the minute hand be 90 degrees ahead of the hour hand?"

Starting off at 3 pm again, the minute hand will be 90 degrees ahead of the hour hand when:

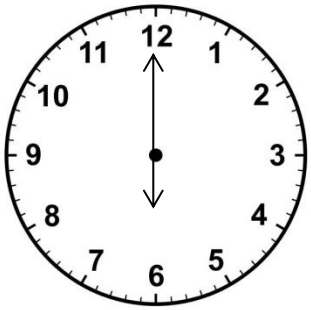
$M = H + 90$
Substituting: $6t = 90 + \frac{1}{2}t + 90$
So that: $5.5t = 180$
$t = 32.727272\ldots$
$= 32$ min 43.6 sec approx.



Again starting from 3 pm, we might ask,
"***When will the hands be at 180 degrees***?"

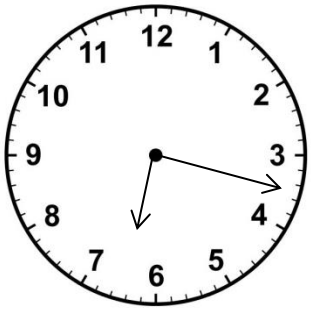
Starting off at 3 pm, the minute hand will be 180 degrees ahead of the hour hand when: $M = H + 180$
Substituting: $6t = 90 + \frac{1}{2}t + 180$
So that: $5.5t = 270$
$t = 49.090909\ldots = 49$ min 5.5 sec approx.

Let us start off at the time of 6 pm



Obviously, the angle between the hands is 180 degrees.

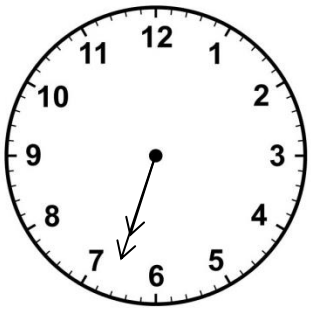
Let's find when the angle between the hands will first be 90 degrees.



As before M = 6t but this time,
H = ½ t + 180 because it starts off at 6
If the angle between them is 90 degrees then:

$$H = M + 90$$
$$\tfrac{1}{2}t + 180 = 6t + 90$$
$$90 = 5.5t$$

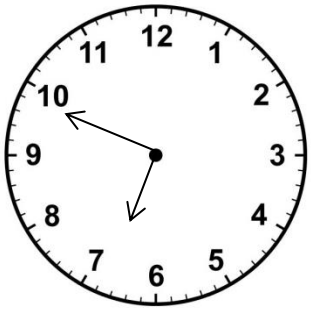
so t = 16.3636….min
= 16 min 21.8 sec approx.



Starting off again at 6 pm now let's ask, "When will the hands be **in line** again?"

$$H = M$$
$$\tfrac{1}{2}t + 180 = 6t$$
$$180 = 5.5t$$

t = 32.727272…
= 32 min 43.6 sec approx.

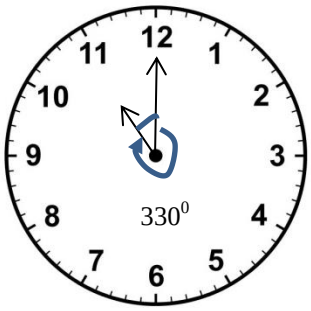


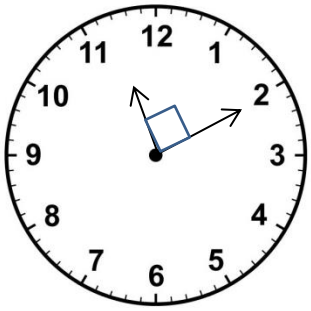
Now let's ask "When will the minute hand be 90 degrees ahead of the hour hand?"

$$M = H + 90$$
$$6t = \tfrac{1}{2}t + 180 + 90$$
$$5.5t = 270$$

t = 49.090909…
= 49 min 5.5 sec approx.

<u>Starting off from 11pm</u>, when will the hands first be at 90 degrees?





In this case M = 6t and H = 330 + ½ t

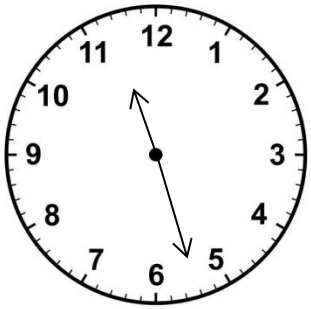
but we need to call it H = – 30 + ½ t
(because we are not dealing with the reflex angle)

The equation to solve is:

M = H + 90

6t = – 30 + ½ t + 90
5.5t = 60
t = 10.909090…
= 10 min 54.5 sec approx..

"Starting off from 11pm, when will the hands first be in line?"



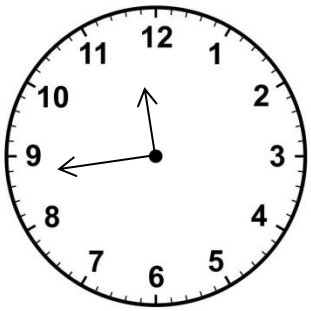
M = 6t and H = – 30 + ½ t

The equation to solve is:

M = H + 180

6t = – 30 + ½ t + 180
5.5t = 150
t = 27.272727…
= 27 min 16.4 sec approx..

Finally, starting off from 11pm, when will the hands next be at 90 degrees?



M = 6t and H = – 30 + ½ t

The equation to solve is:

M = H + 270

6t = – 30 + ½ t + 270
5.5t = 240
t = 43.636363……
= 43 min 38.2 sec. approx..