**SUPERB INVESTIGATION SUITABLE AT MANY LEVELS.**

**(Teacher’s guide)**

1. Consider a piece of string 40 cm long.

 (a) If we make it into a square its area is obviously 10×10 =100 cm2

 10

 10 10

 10

(b) If we cut the string into two pieces 4 cm and 36 cm and form two squares

 find the total area.

 1 9

 1

 9

 Total area = 1 + 81 = 82 cm2

(c) If we cut the string into two pieces 8 cm and 32 cm and form two squares

 find the total area.

 Total area = 2×2 + 8×8 = 68 cm2

(d) Now cut the string into pieces 12 cm and 28 cm.

 Total area = 3×3 + 7×7 = 58 cm2

(e) Make a table of values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1st piece | 2nd piece | Area 1st square | Area 2nd square  | TOTAL AREA |
| 4 | 36 | 1 | 81 | 82 |
| 8 | 32 | 4 | 64 | 68 |
| 12 | 28 | 9 | 49 | 58 |
| 16 | 24 | 16 | 36 | 52 |
| 20 | 20 | 25 | 25 | 50 |
| 24 | 16 | 36 | 16 | 52 |
| 28 | 12 | 49 | 9 | 58 |
| 32 | 8 | 64 | 4 | 68 |
| 36 | 4 | 81 | 1 | 82 |

(f) If the first piece is length ***x***, the other length is ***40 – x***

 Find a formula for the total area:

 ***A = x 2 + (40 – x) 2***

 ***4 4***

 ***= x2 + 1600 – 80x + x2***

 ***16***

 ***= 2x2 – 80x + 1600 or x2 – 5x + 100***

 ***16 8***

Draw a graph of this (possibly using Autograph or a Graphics Calculator)

Point out that this is a parabola. (The “right-way- up” type, so it has a minimum value not a maximum.)

Notice in particular that the minimum value is A = 50 when ***x*** = 20.



Min(20, 50)

2. Consider a piece of string 40 cm long.(*HARDER*)

(a) If we make it into a CIRCLE we need to find the RADIUS, in order to

 calculate its area.

 Students need to know C = 2πr

 r so that 2πr = 40

 πr = 20

 r = 20 ≈ 6.366

 π

 Area = πr2 = π × (6.366)2 ≈ 127.3 cm2

 (b) If we cut the string into two pieces 4 cm and 36 cm and form two circles

 find the total area.

Radius of 1st circle = 2 ≈ 0.6366 Radius of 2nd circle = 18 ≈ 5.73

 π π

Area of 1st circle = π×(0.6366)2 Area of 2nd circle = π×(5.73)2

 = 1.273 cm2 = 103.1 cm2

Total area = 104.4 cm2

(c) If the first piece is length ***x***, the other length is (***40 – x)***

 Find a formula for the total area: (*Quite complicated for students!)*

|  |  |
| --- | --- |
| ***1st Circle******Circumference = x = 2πr*** ***so r = x*** ***2π*** ***Area = πr2 = π × x2 = x2*** ***4 π2 4 π*** | ***2nd Circle******Circumference = 40 – x = 2πr*** ***so r = 40 – x***  ***2π*** ***Area = πr2 = π × (40 – x)2 = (40 – x)2*** ***4 π2 4 π*** |

Total Area = ***x2 + (40 – x)2***

 ***4 π*** ***4 π***

If we draw this graph we do not have to go through lots of ***x*** values to get to the minimum area.



Min (20, 63.66)

Point out that this is a parabola. (The right way up type, so it has a minimum value not a maximum.)

Notice in particular that the minimum value is A = 63.66 when ***x*** = 20.

3. ***(This is where this investigation has been leading.)***

A 40 cm piece of string is cut into 2 pieces.

One piece is made into a CIRCLE and the other is made into a SQUARE.

Find the minimum AREA.

Let one piece be of length ***x*** and the other is (***40 – x)***

Let the 1st piece be made into a circle:

***Circumference = x = 2πr***

***so r = x***

 ***2π***

***Area = πr2 = π × x2 = x2***

 ***4 π2 4 π***

Let the 2nd piece be made into the square:

***Each side will be 40 – x***

 ***4***

***So the area is (40 – x )2***

 ***16***

Now we will draw the graph Area ***y*** = ***x2 + (40 – x )2***

 ***4 π 16***

 

Min (17.6, 56)

Also notice that when ***x*** = 40 only a circle is made and its area is 127.3 cm2

Notice when

***x*** = 0

only a square is made and its area is 100 cm2

 The minimum Area is 56.01 cm2 when ***x*** = 17.6 cm

Although we have used a graph program to find this minimum value, this makes a good calculus problem.

***Lengths are x and (40 – x). If circumference = x then 2πr = x so r = x***

 ***(2π)***

***Total Area = πr2 + (40 – x)2***

 ***16***

 ***AREA = π x2 + 1600 – 80x + x2 = x2 + 100 – 5x + x2 40 – x***

 ***4π2 16 4π 16 4***

 ***d(Area) = x – 5 + x = 0 for max Area***

 ***dx 2π 8 40 – x***

 ***4***

***x( 1 + 1) = 5***

 ***2π 8***

***x ×0.28415 = 5***

 ***x = 17.6 cm so r = 2.8 cm***

***So Min Area = π×2.82 + 5.62 = 56 cm2***

A nice extension to this would be splitting the string into 2 pieces, making a SQUARE with one piece and an EQUILATERAL TRIANGLE with the other.

 ***x x***

 ***x***

If we let each side of the triangle be ***x*** then the area could simply be written as ***x × x × sin60 = x2 √3***

 ***2 4***

This leaves (***40 – 3x)*** left for the square so each side = ***40 – 3x***

 ***4***

The area of the square = ***(40 – 3x)2 = 1600 – 240x + 9x2***

 ***16 16***

The total area is A = ***x2 √3 + 1600 – 240x + 9x2***

 ***4 16***

***dA = x√3 – 15 + 9x = 0 at min area***

***dx 2 8***

***x(√3 + 9 ) = 15***

 ***2 8***

***x ≈ 7.53***

Min A ≈ 43.5 cm2



Min (7.53, 43.5)