**THE FUNDAMENTAL THEOREM OF CALCULUS.**

There are TWO different types of **CALCULUS**:

1. **DIFFERENTIATION:** finding **gradients** of curves.

2. **INTEGRATION:** finding **areas** under curves.

To estimate this area: We could split it into strips as below:



We neglect these little “triangular” bits and treat them as rectangles.



To get better approximations we We can see the approximation

could split the area into more strips: gets better and better as we use

 more and more strips:

The little “triangular” bits become more and more negligible.





The sum of the areas of these strips gets closer and closer to the actual area under the curve. We can find this limit as follows:

Consider one strip greatly enlarged for clarity.

We will neglect the curved triangular bit on the top and treat the strip as a rectangle of height ***f(x)*** and width ***h***.

Area of 1 strip ≈ ***f(x) × h***

 ***f(x)***

Area of all strips ≈ ∑ ***f(x) × h***

 ***h***

Actual area under the curve = ***lim ∑ f(x)×h*** which is written as **∫ *f(x) dx***

 h→0

NB At this stage the sign **∫** only means **the limit of the sum of the strips.**

There is no indication yet that **∫** has anything to do with **antidifferentiation.**

**Suppose there exists a “formula” or expression, in terms of *x*, to find the area**. (just like there is a formula to find the area of a circle = πr2.)

**We will call this formula or function *A(x).***

**It is most helpful to think of this in the following way:**

***A(x1)*** = area under the curve from ***x = 0 to x = x1***



 0 ***x1***

***A(x1)***



 0 ***x2***

***A(x2)*** = area under the curve from ***x = 0 to x = x2***

***A(x2)***



 ***x1 x2***

So the area from ***x1 to x2*** could

be written as ***A(x2) – A(x1)***

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*Now consider just one of these strips mentioned earlier. (greatly enlarged)

 F E

The area of the strip could be written as : D C

 ***A(x+h) – A(x)***

 ***f(x) f(x+h)***

The area of the **strip** is between the areas of the

rectangles ABCD and ABEF:

**area ABCD** **<** ***A(x + h) – A(x)*** **<** **area ABEF**

 ***h***

 ***f(x)×h < A(x + h) – A(x) < f(x + h)×h*** A B

Dividing throughout by ***h***, we get:

  ***f(x) < A(x+h) – A(x) < f(x+h)***

 ***h***

If we find the **limit** of this as ***h*** 0, the three quantities become equal because the two outer quantities both become ***f(x).***

 ***lim f(x) < lim A(x+h) – A(x) < lim f(x+h)***

 ***h→0 h→0  h h→0***

So that: ***f(x) < dA < f(x)***

 ***dx***

 Obviously ***dA = f(x)***

 ***dx***

In words, this says,

“ ***If we* DIFFERENTIATE *the expression for the* AREA *we get the***

**EQUATION *of the curve***.”

Or in other words,

“***The formula for* AREA *is the* ANTIDERIVATIVE *of the***

**equation of the curve*.”***

Finding the área under a curve using “**the limit of the sum of the strips”** as previously explained, is a very complicated algebraic experience but we have just shown that the sign **∫** notonly means **“the limit of the sum of the strips”** but it also means **antidifferentiate.**

 ***b***

 ***a***

 Area A = **∫ *f(x) dx*** **now means**:

***Find the antiderivative of f(x) = A(x) then calculate A(b) – A(a)***

4

2

eg if ***y = f(x) = 3x2*** and ***a = 2*** and ***b = 4*** then Area A = **∫ *3x2 dx***

 4

= ***x3***  = 43 – 23 = 64 – 8 = 56 units2

 2

EDIT. An example of integrating ***y = x2*** from ***x = 0 to x = 3*** by splitting the area into strips is shown below: