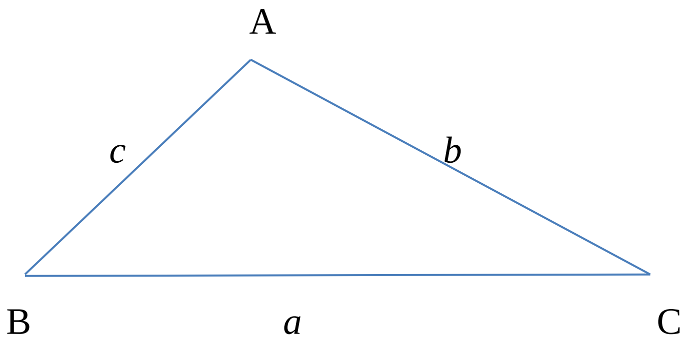
LLOYD'S FORMULA for the area of any triangle with sides *a, b* and *c*.



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Using $\sin^2 C = 1 - \cos^2 C$

$$\sin^2 C = 1 - \frac{(a^2 + b^2 - c^2)^2}{4a^2b^2}$$

$$\sin^2 C = \frac{4a^2b^2 - (a^2 + b^2 - c^2)^2}{4a^2b^2}$$

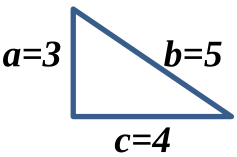
$$\sin C = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{2ab}$$

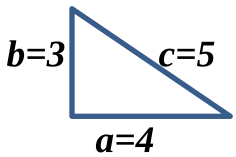
$$\text{Area of } \Delta ABC = \frac{ab\sin C}{2}$$

$$= \frac{ab}{2}\frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{2ab}$$

$$\textbf{AREA} = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$$

Checks (with a, b and c in differing combinations):





$$\textbf{Area} = \frac{\sqrt{(4\times9\times25 - (9+25-16)^2)}}{4}$$

$$= \frac{\sqrt{(900 - 18^2)}}{4}$$

$$= \frac{\sqrt{576}}{4}$$

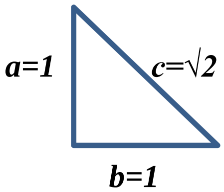
$$= 6$$

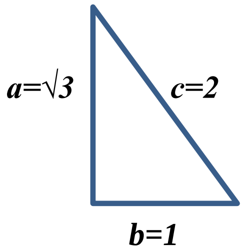
$$\textbf{Area} = \frac{\sqrt{(4\times16\times9 - (16+9-25)^2)}}{4}$$

$$= \frac{\sqrt{(576 - (0)^2}}{4}$$

$$= \frac{\sqrt{576}}{4}$$

$$= 6$$





$AREA = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$

$= \frac{\sqrt{(4 - (1 + 1 - 2)^2)}}{4}$

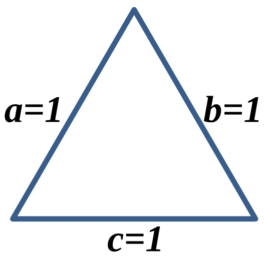
$= ½$

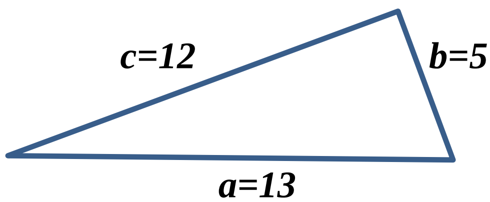
$AREA = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$

$= \frac{\sqrt{(4 \times 3 \times 1 - (3 + 1 - 4)}}{4}$

$= \frac{\sqrt{12}}{4}$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$





$AREA = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$

$= \frac{\sqrt{4 - 1^2}}{4}$

$= \frac{\sqrt{3}}{4}$

$AREA = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$

$= \frac{\sqrt{(16900 - (50)^2)}}{4}$

$= \frac{\sqrt{14400}}{4} = \frac{120}{4} = 30$

ie same as $\frac{bh}{2} = \frac{5 \times 12}{2} = 30$

SPECIAL NOTE:

$AREA = \frac{\sqrt{(4a^2b^2 - (a^2 + b^2 - c^2)^2)}}{4}$

Expanded $= \frac{\sqrt{(a^4 + b^4 + c^4 + 6a^2b^2 - 2a^2c^2 - 2b^2c^2)}}{4}$

and with great effort this could be factorised to make Heron's formula:

Area $= \frac{\sqrt{(a + b + c)(b + c - a)(a + c - b)(a + b - c)}}{4}$

$= \sqrt{\frac{(a+b+c)}{2}\left(\frac{a+b+c}{2} - a\right)\left(\frac{a+b+c}{2} - b\right)\left(\frac{a+b+c}{2} - c\right)}$

$= \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{(a + b + c)}{2}$