WHY DO WE BOTHER WITH RADIANS IN YEAR 12 MATHEMATICS?

The ONLY reason to use radians in preference to using degrees is when we differentiate or integrate trigonometric functions in Year 13 Calculus.

If $y = \sin x$ where x is in **DEGREES**

then $\dfrac{dy}{dx} = 0.01745 \times \cos x \qquad \left(\text{or } \dfrac{\pi}{180} \cos x \right)$

but if $y = \sin x$ where x is in radians

then $\dfrac{dy}{dx} = 1 \times \cos x$

In all other situations such as calculating arc length, area of sectors, sketching graphs, solving trigonometric equations and modelling, there is no real need for radians.

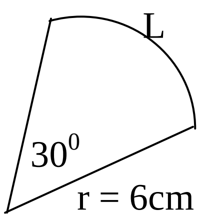
The special "aesthetic quality" of radians is simply a myth!

Both "radians" and "degrees" are really JUST different ways of measuring angles, just as "metres" and "feet" are just different ways of measuring lengths.

The requirement for students to use radians at this level is making mathematics <u>more inaccessible</u> than it needs to be!

Eg

We **DO NOT** need special radian formulae to find **arc length** and **areas of sectors**.



This is simply $\dfrac{30}{360}^{\text{th}}$ or $\dfrac{1}{12}^{\text{th}}$ of a full circle.

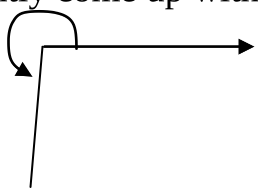
so arc length $L = \dfrac{1}{12} \times \pi d = \dfrac{1}{12} \times \pi \times 12 = \pi$ cm

and Area $A = \dfrac{1}{12} \times \pi r^2 = \dfrac{1}{12} \times \pi \times 36 = 3\pi \text{ cm}^2$

There is never a need to resort to formulae such as $L = r\theta$ or $A = \frac{1}{2} r^2\theta$ when all that is required is simple YEAR 9 LEVEL LOGIC!

My next point is this: WHO REALLY USES RADIANS?

Ask any mathematician or scientist to **visualise an angle of 4.7 rads.**
On the other hand, ask any Year 9 student to visualise an angle of 269^0 and they will confidently come up with an angle as follows :



Now be honest, did YOU know that 4.7 rads is just less than 270^0 ?

When we SAY we are using radians, we are **usually** talking about angles such as:
$\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{3\pi}{2}$, 2π etc

Again, if we are honest, when we are talking about $\frac{\pi}{6}$ radians **we really mean 30^0**,

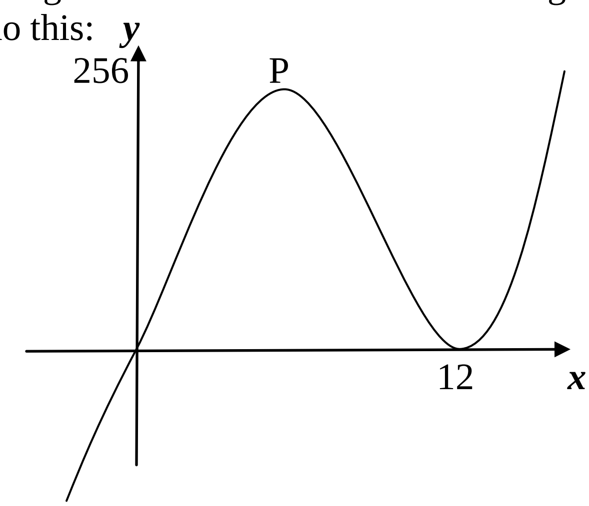
Actually, $\frac{\pi}{6}$ radians is really just 30^0 **in disguise** !!

The actual value of $\frac{\pi}{6}$ is of course 0.523598775… How silly is that?

Similarly $\frac{\pi}{4}$ is really 45^0 , $\frac{3\pi}{2}$ is really 270^0 and 2π radians is really 360^0

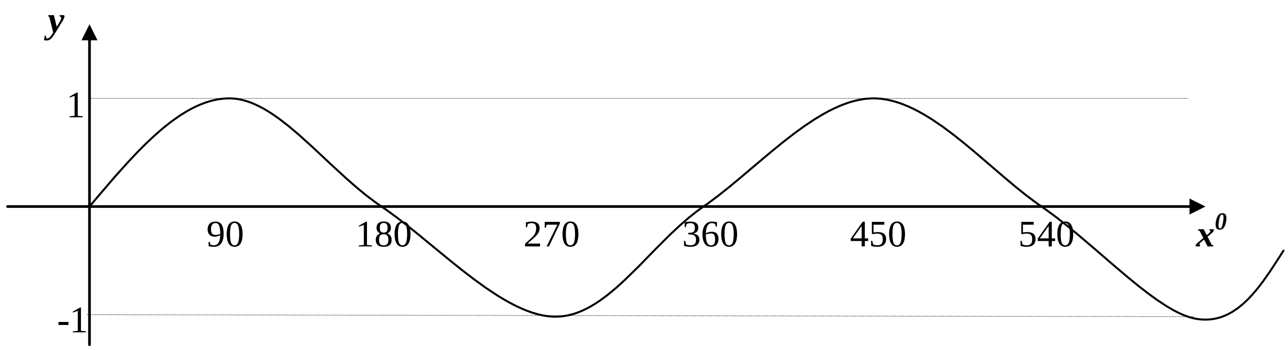
We do not use angles of $\frac{\pi}{7}$ for instance, **because it has no nice equivalent in degrees**!

Some people say we use radians so that the x and y scales on the trig graphs are more even. Let's get real! When we sketch a cubic graph such as ***y* = *x*(*x* – 12)^2**
we just do this:

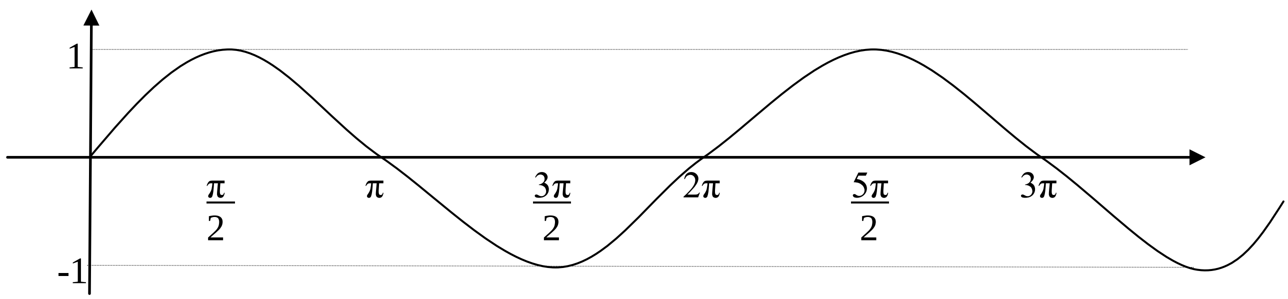


We do not concern ourselves that the scales are not even!
The coordinates of P are (4, **256**) ?

Similarly the graph of $y = \sin x$, where x is in degrees, is fine just the way it is.
The scales on x and y axes **DO NOT** have to be the same.

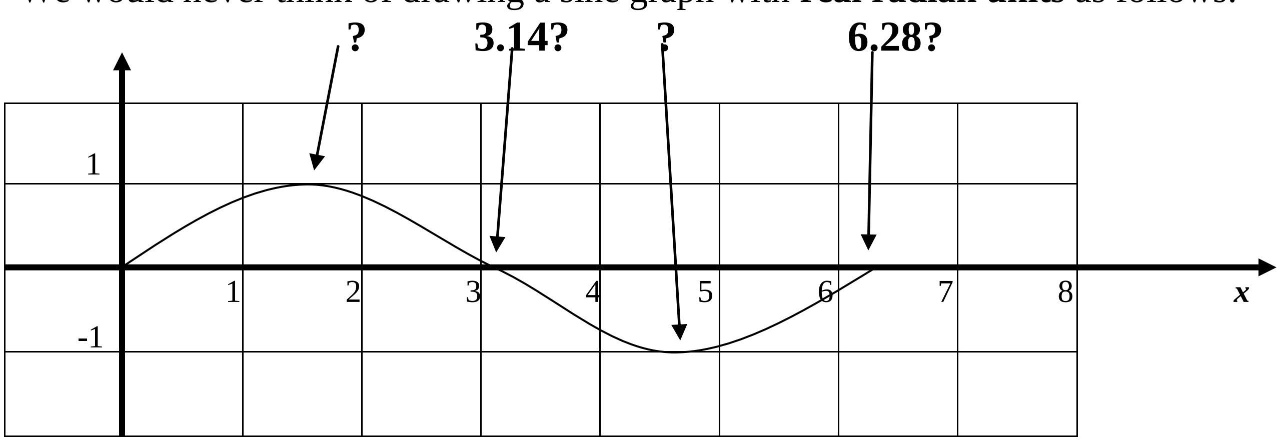


<u>Now here is a very interesting point</u>.
When we draw a sine graph with a "radian scale", this is what we draw:



THIS IS AN ABSOLUTE FRAUD!
We are really marking the special points as they occur **in degrees**.

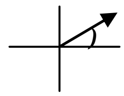
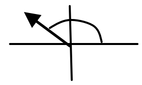
We would never think of drawing a sine graph with **real radian units** as follows:



The intercepts on the x axis and positions of max/min points are not at all obvious!

<u>When solving simple trig equations, we can only understand and visualise the size of our answers when we use degrees.</u>

Eg If $\boldsymbol{\sin 2x = \frac{1}{2}}$
$\boldsymbol{in\ 0 \le x \le 360^0}$

then $\boldsymbol{2x = 30,\ 150,\ 390,\ 510}$
$\boldsymbol{so\ x = 15,\ 75,\ 195,\ 255}$

All these answers MEAN something to us.
We understand their SIZE.
We can see they are in the interval $\boldsymbol{0 \le x \le 360^0}$

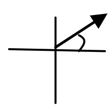
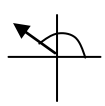
Now consider the same thing in radians:

If $\boldsymbol{\sin 2x = \frac{1}{2}}$ $\boldsymbol{in\ 0 \le x \le 2\pi}$

Then $\boldsymbol{2x = 0.5236,\ 2.618,\ 6.8068,\ 8.9012}$
$\boldsymbol{so\ x = 0.262,\ 1.309,\ 3.403,\ 4.45}$

Do even mathematicians find these numbers meaningful or helpful?
Have we any idea of the size of these angles?
Are they obviously in the interval $\boldsymbol{0 \le x \le 2\pi}$ **?**
Have we missed any out ?

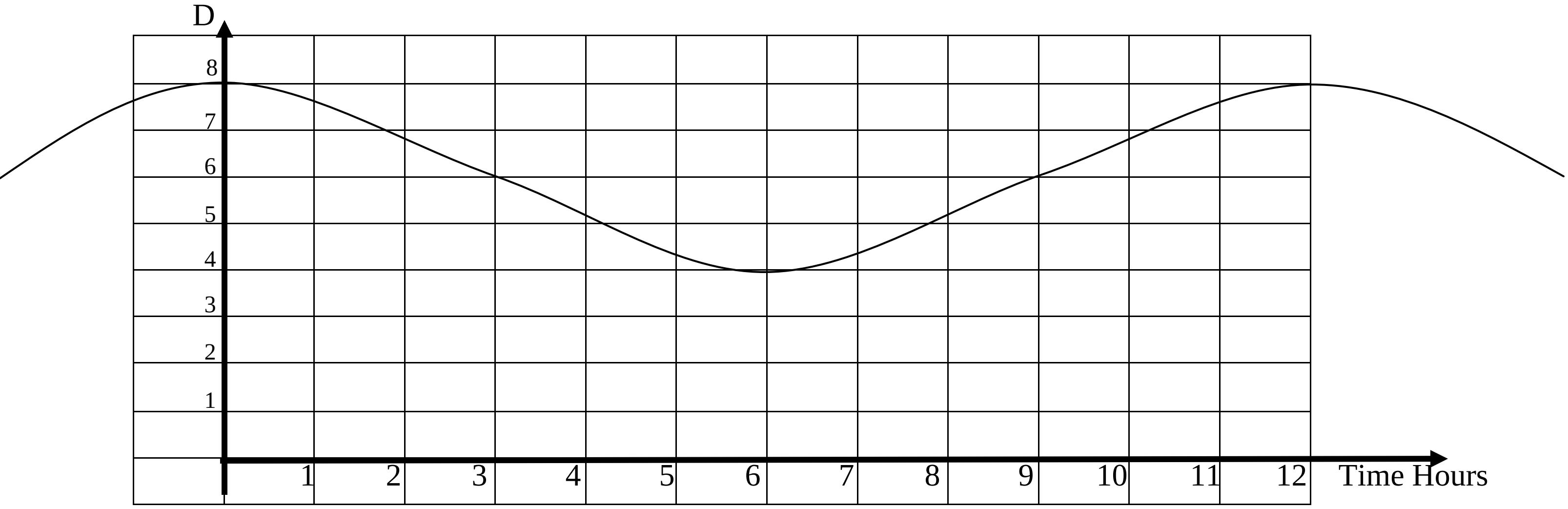
Consider equations like: $\boldsymbol{\sin(x + b) = c}$

Using DEGREES in $\boldsymbol{0 \le x \le 360^0}$	Using RADIANS in $\boldsymbol{0 \le x \le 2\pi}$
$\boldsymbol{Sin(x + 60^0) = 0.5}$  	$\boldsymbol{Sin(x + \pi/3) = 0.5}$
$\boldsymbol{x + 60 = 30\ \ or\ \ x + 60 = 150}$	$\boldsymbol{x + \pi/3 = \pi/6\ \ or\ x + \pi/3 = 5\pi/6}$
$\boldsymbol{x = -30\ \ or\ \ x = 90}$	$\boldsymbol{x = -\pi/6\ \ or\ \ \pi/2}$
$\boldsymbol{x = 330\ \ or\ \ 90}$	$\boldsymbol{x = 11\pi/3\ \ or\ \pi/2}$

Again the **<u>use of radians greatly increases the difficulty level</u>** of what seems to be a simple, logical, meaningful procedure when using degrees.

<u>Now we come to things that can be modelled using trigonometrical functions.</u>

1. Suppose the tide is high every 12 hours. Let the max depth at Queen's wharf be 8m at 12 am, and the min depth be 4m.



The situation can be modelled by an equation of the form $D = a + b\cos(ct)$

We can simply find that $a = 6$, $b = 2$.

The period is when $ct = 360^0$

$$c \times 12 = 360$$
$$c = 30$$

The equation of the model is $D = 6 + 2\cos(30t)$

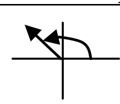
In this model, *t* is in HOURS but (*30t*) is in degrees

If we were to use radians, the formula would have been $D = 6 + 2\cos(0.5236\,t)$

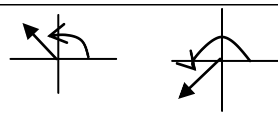
In this model, *t* is still in HOURS but *(0.5236 t)* is in radians.

There is absolutely NO DOUBT in my mind as to which of these model equations is more meaningful.

Eg What is the first time the depth will be 5 metres?

<u>Degrees</u>	**<u>Radians</u>**
	
$5 = 6 + 2\cos(30t)$	$5 = 6 + 2\cos(0.5236\,t)$
$\cos(30t) = -½$	$\cos(0.5236\,t) = -½$
$30t = 120$	$0.5236\,t = 2.0944$
so t = 4 hours after midnight = 4 am	*so t = 4 hours after midnight = 4 am*
	(an obvious source of rounding error here!)

Suppose an ocean liner can only be at the wharf if the tide is over 4.5 metres deep.
Find the times in the first 12 hours after midnight, that the liner cannot be at the wharf.

Degrees	**Radians**
 $4.5 = 6 + 2\cos(30t)$ $\cos(30t) = -0.75$ $30t = 138.6\ ,\ 221.4$ *so t = 4.62 and 7.38 hours after midnight* *= 4:37 am and 7:23 am*	$4.5 = 6 + 2\cos(0.5236\, t)$ $\cos(0.5236\, t) = -0.75$ $0.5236\, t = 2.419\ ,\ 3.864$ *so t = 4.62 and 7.38 hours after midnight* *= 4:37 am and 7:23 am*

Conclusion, the liner cannot be at the wharf between 4:37 am and 7:23 am.

NB If an actual tide is high every 12.5 hours then using the general equation:

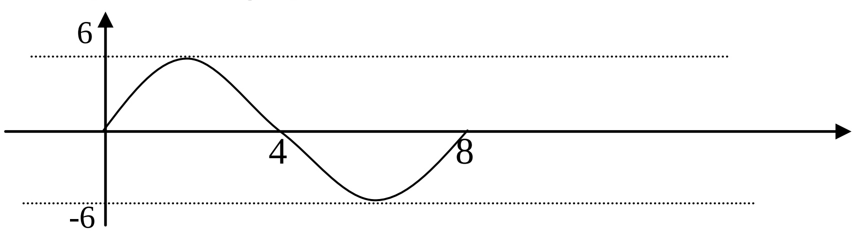
$D = \cos(bt)$

1 period is when $b \times 12.5 = 360$

so $b = 28.8$

the equation would be $D = \cos(28.8t)$

When finding the equation of graphs such as this one :



The amplitude is 6 so obviously we could say $y = 6\sin(bt)$

USING DEGREES

One period is when $bt = 360^0$

$b \times 8 = 360$

$b = 45$

The equation is:

$y = 6\sin(45t)$

USING RADIANS

One period is when $bt = 2\pi$ *rads*

$b \times 8 = 2\pi$

$b = \frac{\pi}{4}$ or 0.7854

The equation is:

$y = 6\sin(0.7854t)$

NCEA LEVEL 2. Excellence Question)

The loudness of a car alarm can be modelled by :

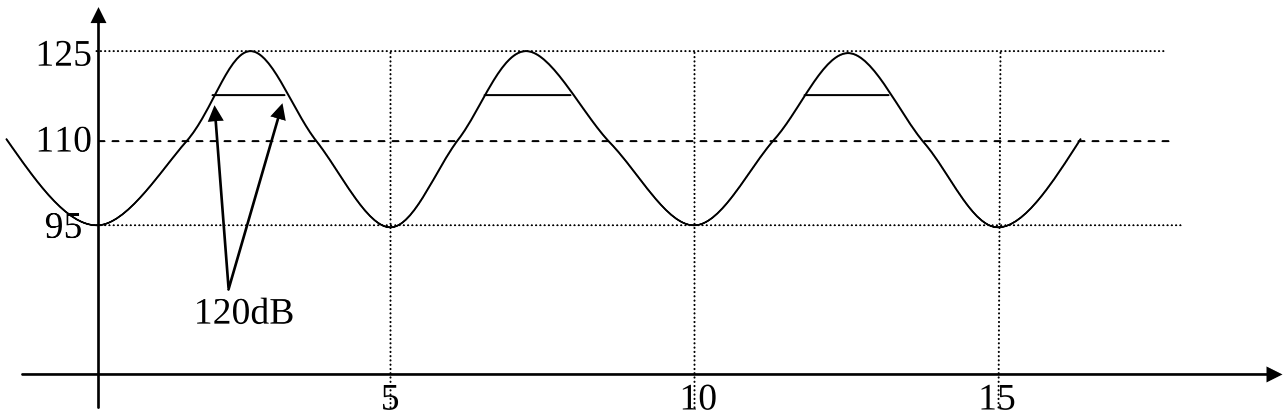
L = 110 – 15cos(72t) L = loudness in decibels t = time in seconds

Find the total amount of time in the 1st 15 secs that the loudness is over 120 decibels.

(If it is > 6 sec the alarm cannot be turned off without hurting the owner's ears.)

1 period is when 72t = 360 so period t = 5 sec (NB 15 sec is 3 complete periods)

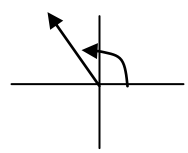
Max L is 110 + 15 = 125 Min L is 110 – 15 = 95

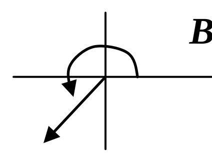


If L = 120 then 120 = 110 – 15cos72t

So 15cos 72t = -10

$$\cos 72t = -\frac{10}{15}$$





Basic angle = 48.2⁰

72t = 131.8 , 228.2 so t = 1.83 sec , 3.17 sec

During the 1st period, the time that the loudness L is > 120 dB is 1.37 – 1.83 = 1.34 sec

This will be the same for each period and so the total time during the 1st 3 periods will be 1.34 × 3 = 4.02 sec. so the alarm can be turned off safely because the time that L > 120 during 1st 15 sec is definitely less than 6 sec.

CONCLUSION.

The problems at NCEA level 2 and 3 for Trigonometrical Equations would be much more meaningful and accessible to students if **degrees** were used rather than **radians**. But, what is far more important, **more students would understand the topic much better if we left radians alone until we need them just for differentiation in Year 13 Calculus**.

Philip Lloyd