

## DIVISION OF FRACTIONS.

I find that even Year 13 students (17 and 18 year olds) do not know **WHY** we “turn the 2<sup>nd</sup> number upside down and multiply”!

A **simple** explanation for 12 and 13 year olds (Year 9) is this:

**We know that  $6 \div 2 = 3$  and  $6 \times \frac{1}{2} = 3$**

**so DIVIDING by 2 is the same as MULTIPLYING by  $\frac{1}{2}$**

We can give “explanations” ad nauseam but frankly I find that these just go way over the heads of young students.

The following idea is probably the limit of what we can realistically expect students to follow confidently:

**Think about 6 divided by  $\frac{1}{2}$**

(Most people, including adults, would instinctively say  $6 \div \frac{1}{2} = 3!!!$  because they mistakenly think of it as “a half of 6” = 3 )

A good idea is to say  $6 \div \frac{1}{2}$  means “how many halves are there in 6”?  
The answer is of course 12

If  $6 \div \frac{1}{2} = 12$  this is the same as  $6 \times \frac{2}{1} = 12$

**Putting these SIMPLE cases together we can see that:**

**DIVIDING by 2 is the same as MULTIPLYING by  $\frac{1}{2}$**

**DIVIDING by  $\frac{1}{2}$  is the same as MULTIPLYING by  $\frac{2}{1}$**

Using these examples we can get to the idea quickly that to divide we **turn the 2<sup>nd</sup> number “upside down” and multiply.**

(“Turning a number upside down” is finding the “reciprocal”.)

So, to **divide** by 5 we **multiply** by  $\frac{1}{5}$

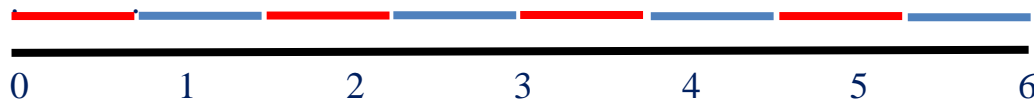
At this stage, young students will happily accept the idea that in order to divide by  $\frac{3}{4}$  we multiply by  $\frac{4}{3}$

Many teachers go too far in their efforts to get young students to understand. I think the following is a little too far:

Consider 6 divided by  $\frac{3}{4}$

This means “**how many three quarters are there in 6**”?  
(The answer is not at all easy for students to visualise even if it seems easy for teachers!)

A diagram could help...



Using **rods** we can see the answer is **8 lots of  $\frac{3}{4}$  make up 6.**

It **does NOT help** if teachers use language with statements like:  
“**Dividing by a fraction is just the same as multiplying by its reciprocal!**”

The proper setting out should be:

$$\begin{aligned} & \frac{5}{7} \div \frac{3}{4} \\ &= \frac{5}{7} \times \frac{4}{3} \\ &= \frac{20}{21} \end{aligned}$$

Although the following is not an actual “proof” but for older students (16 years and over) this working definitely shows WHY we “**turn the second fraction upside down and the multiply**”!

$$\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}$$

rewrite the division in the form  $\frac{a}{b}$

$$\frac{5}{7} \times \frac{1}{1}$$

multiplying by  $\frac{1}{1}$  does not change the fraction

$$\frac{5}{7} \times \frac{4}{3}$$

$\frac{4}{3}$  is just the same as  $\frac{1}{1}$

$$\frac{5}{7} \times \frac{4}{3}$$

$\frac{3}{4} \times \frac{4}{3}$   
 Cancelling out the  $\frac{3}{4} \times \frac{4}{3} = 1$   
 in the denominator:

$$\frac{5}{7} \times \frac{4}{3}$$

This is equivalent to turning the second fraction upside down and then multiplying.

$$= \frac{20}{21}$$

Here is a short video covering the above ideas...

<http://screencast.com/t/XIESJ3yEqfzJ>