DOES $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

It is all about the ORDER in which the calculations are done.

$\sqrt{a \times b}$ means multiply $\boldsymbol{a}$ and $\boldsymbol{b}$ THEN find the square root.
$\sqrt{a} \times \sqrt{b}$ means find each square root THEN multiply them.

Sometimes they are not equal!

These examples will show when.

Consider $\sqrt{\mathbf{4}} \times \sqrt{\mathbf{9}} = \mathbf{2} \times \mathbf{3} = \mathbf{6}$
and $\sqrt{(\mathbf{4} \times \mathbf{9})} = \sqrt{\mathbf{36}} = \mathbf{6}$

So when both numbers are positive $\sqrt{\boldsymbol{a} \times \boldsymbol{b}} = \sqrt{\boldsymbol{a}} \times \sqrt{\boldsymbol{b}}$

Now consider $\sqrt{\mathbf{4}} \times \sqrt{(-\mathbf{9})} = \mathbf{2} \times \mathbf{3}\boldsymbol{i} = \mathbf{6}\boldsymbol{i}$
and $\sqrt{\mathbf{4} \times (-\mathbf{9})} = \sqrt{-\mathbf{36}} = \mathbf{6}\boldsymbol{i}$

So when one number is positive and the other is negative $\sqrt{\boldsymbol{a} \times \boldsymbol{b}} = \sqrt{\boldsymbol{a}} \times \sqrt{\boldsymbol{b}}$

Now consider $\sqrt{(-\mathbf{4})} \times \sqrt{(-\mathbf{9})} = \mathbf{2}\boldsymbol{i} \times \mathbf{3}\boldsymbol{i} = -\mathbf{6}$
BUT $\sqrt{(-\mathbf{4}) \times (-\mathbf{9})} = \sqrt{\mathbf{36}} = +\mathbf{6}$

So when both numbers are **negative** $\sqrt{\boldsymbol{a} \times \boldsymbol{b}} \neq \sqrt{\boldsymbol{a}} \times \sqrt{\boldsymbol{b}}$

This is the source of many **false proofs** for example:

$$\mathbf{1} = \sqrt{+\mathbf{1}} = \sqrt{(-\mathbf{1}) \times (-\mathbf{1})} = \sqrt{(-\mathbf{1})} \times \sqrt{(-\mathbf{1})} = \boldsymbol{i} \times \boldsymbol{i} = -\mathbf{1}$$

This **seems** to prove **1 = –1** but the **false step** is in **RED** type!

We should also consider if $\sqrt{\frac{\boldsymbol{a}}{\boldsymbol{b}}} = \frac{\sqrt{\boldsymbol{a}}}{\sqrt{\boldsymbol{b}}}$

$\sqrt{\frac{a}{b}}$ means "find ***a*** divided by ***b*** first **then** find the square root"

while $\frac{\sqrt{a}}{\sqrt{b}}$ means "find the square roots of ***a*** and ***b*** first **then** divide them".

Example 1.

$\sqrt{\frac{64}{16}} = \sqrt{4} = 2$	$\frac{\sqrt{64}}{\sqrt{16}} = \frac{8}{4} = 2$
So $\sqrt{\frac{64}{16}} = \frac{\sqrt{64}}{\sqrt{16}}$	

Example 2.

$\sqrt{\frac{-64}{16}} = \sqrt{-4} = 2i$	$\frac{\sqrt{-64}}{\sqrt{16}} = \frac{8i}{4} = 2i$
So $\sqrt{\frac{-64}{16}} = \frac{\sqrt{-64}}{\sqrt{16}}$	

Example 3.

$\sqrt{\frac{64}{-16}} = \sqrt{-4} = \boldsymbol{2i}$	$\frac{\sqrt{64}}{\sqrt{-16}} = \frac{8}{4i} = \frac{8}{4i} \times \frac{i}{i} = \boldsymbol{-2i}$
So $\sqrt{\frac{64}{-16}} \neq \frac{\sqrt{64}}{\sqrt{-16}}$	

Example 4.

$\sqrt{\frac{-64}{-16}} = \sqrt{4} = \boldsymbol{2}$	$\frac{\sqrt{-64}}{\sqrt{-16}} = \frac{8i}{4i} = \boldsymbol{2}$
So $\sqrt{\frac{-64}{-16}} = \frac{\sqrt{-64}}{\sqrt{-16}}$	