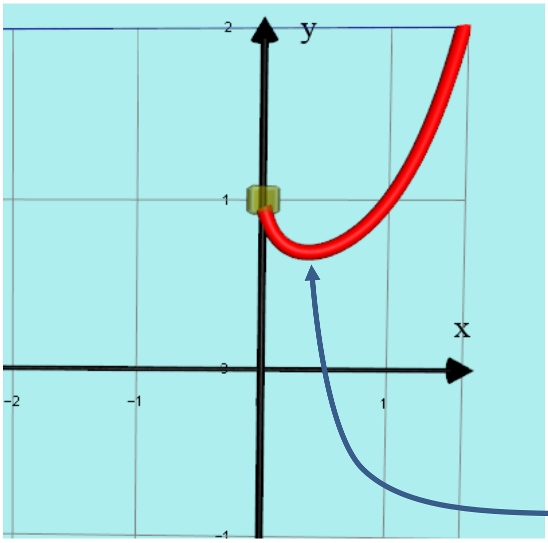
The graph of y = x^x

I have been working on this graph and I found the graph to be ABSOLUTELY fascinating.

For $x > 0$ there is no problem.

It looks like this:



If $y = x^x$

then $\ln(y) = \ln(x^x)$

so that $\ln(y) = x\ln(x)$

therefore differentiating:

we get $\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + 1 \times \ln(x)$

$= 1 + \ln(x)$

If the gradient is zero then $\ln(x) = -1$

so $x = e^{-1} \approx 0.367879$ ***and*** $y = 0.6922$

This is a minimum point.

Many people believe that the minimum value of x^x is 0.6922…

but there is a <u>left hand side</u> to the graph when $x < 0$ where it gets very exciting!

If $x < 0$ we CAN find y values!

For instance if:

$x = -1$ we get $(-1)^{(-1)} = -1$

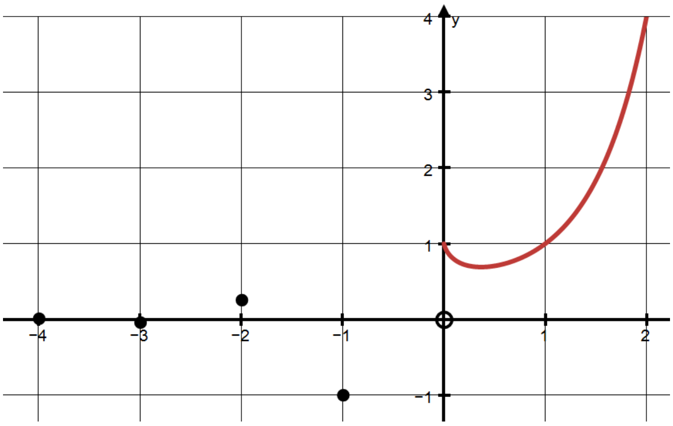
$x = -2$ we get $(-2)^{(-2)} = +\frac{1}{4}$

$x = -3$ we get $(-3)^{(-3)} = -0.037$

$x = -4$ we get $(-4)^{(-4)} = +0.0039$

These are all REAL numbers and they are less than 0.6922!

If we plot these extra points we get something like this…



…and these are not just a few isolated points! I will explain below…

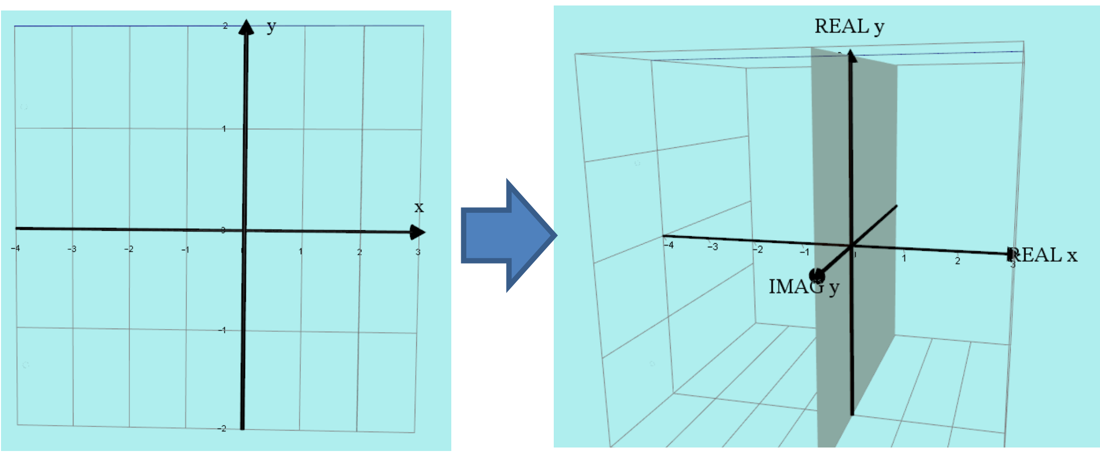
If x = –0.5 then y = (–0.5)^(–0.5) = ***– 1.414i***

If x = –1.5 then y = (–1.5)^(–1.5) = ***+0.544i***

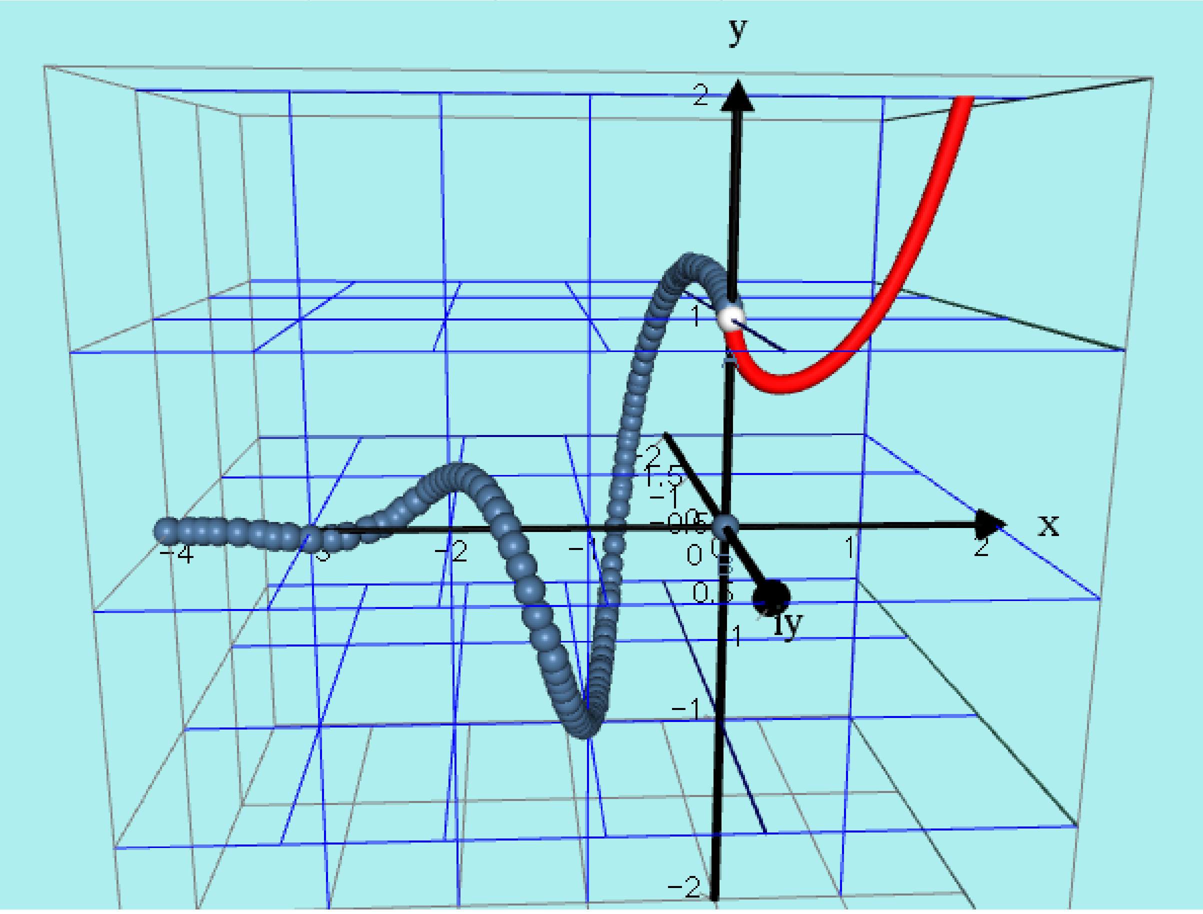
If x = –1.6 then y = (–1.6)^(–1.6) = ***0.15 + 0.45i***

We can plot these points if we change from the normal x, y axes to a REAL x axis sticking through a complex y plane as below:

x, y axes　　　　Real x axis and **complex y PLANE**



Now if we plot **many** points we get this amazing shape!



I finally found the equation of the actual spiral and produced this wonderful graph…