Why is $\sqrt[3]{-1} \neq -1$

Now that more people are getting better calculators they start to re-think what they always thought was obvious!

These new calculators give what we call the PRINCIPAL ROOT or PRIMARY ROOT.

It may sound a bit pedantic but $\sqrt{9} = 3\ but\ not - 3$

However if we solve $x^2 = 9$ we must write $x = \pm\sqrt{9} = \pm 3$
We need the ± sign because the root sign √ **only** means the positive answer.

In fact $\sqrt[3]{8}$ only means the principal root which is 2.
but we do know there are 3 roots of $x^3 = 8$

I like the simple De Moivre method…

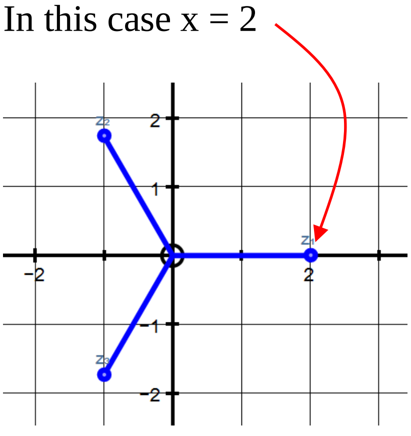
$$x^3 = 8$$
$$[rcis(\theta)]^3 = 8cis(0 + 360n)$$
$$r^3 cis(3\theta) = 8cis(0 + 360n)$$

$$r^3 = 8\ so\ r = 2\ and\ 3\theta = 360n\ so\ \theta = 120n = 0, 120, 240$$

The 3 solutions are:

$$\boldsymbol{x_1 = 2cis(0) = 2\cos(0) + 2isin(0) = 2}$$
$$x_2 = 2cis(120) = 2\cos(120) + 2isin(120) = -1 + i\sqrt{3}$$
$$x_3 = 2cis(240) = 2\cos(240) + 2isin(240) = -1 - i\sqrt{3}$$

The principal root is the first one found by De Moivre's Theorem.
In this case x = 2



However, if …

$$x^3 = -8$$
$$[rcis(\theta)]^3 = 8cis(180 + 360n)$$
$$r^3cis(3\theta) = 8cis(180 + 360n)$$

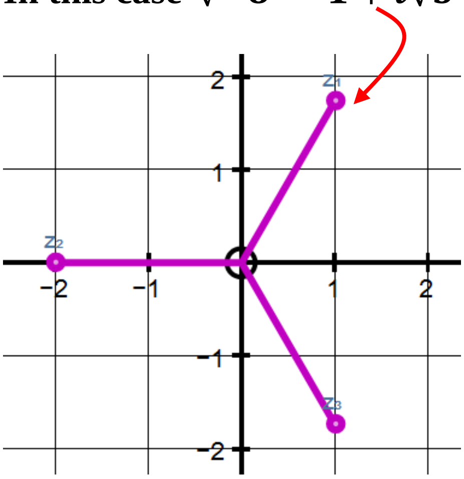
$$r^3 = 8 \; so \; r = 2 \; and \; 3\theta = 60 + 360n \; so \; \theta = 60, 180, 300$$

The 3 solutions are:

$$\boldsymbol{x_1 = 2cis(60) = 2\cos(60) + 2isin(60) = 1 + i\sqrt{3}}$$
$$x_2 = 2cis(180) = 2\cos(180) + 2isin(180) = -2$$
$$x_3 = 2cis(300) = 2\cos(300) + 2isin(300) = 1 - i\sqrt{3}$$

The **principal root** is the first one found by De Moivre's Theorem.

In this case $\sqrt[3]{-8} = 1 + i\sqrt{3}$



A good case is when NONE of the answers is real!

$$x^4 = -1$$
$$[rcis(\theta)]^4 = 1cis(180 + 360n)$$
$$r^4 cis(4\theta) = 1cis(180 + 360n)$$

$$r^4 = 1 \; so \; r = 1 \; and \; 4\theta = 180 + 360n \; so \; \theta = 45, 135, 225, 315$$

The 4 solutions are:

$$\boldsymbol{x_1 = 1cis(45) = 1\cos(45) + 1isin(45) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}}$$
$$x_2 = 1cis(135) = 1\cos(135) + 1isin(135) = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$
$$x_3 = 1cis(225) = 1\cos(225) + 1isin(225) = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$$
$$x_4 = 1cis(315) = 1\cos(315) + 1isin(315) = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$$

So $\sqrt[4]{-1} = \boldsymbol{\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}}$