What topic of math should I study before understanding the binomial theorem?

Actually, there is very little you really NEED to know before you can expand binomial expressions.
Some people who rely on formulas to do everything, may say you need to know about Pascal's triangle and Combination/Permutation theory.

But there is NO NEED to use the ${}^nC_r$ notation AT ALL.

I never use this!

I call the following way to expand binomial expressions the "**thinking method**" as opposed to the **"formula method**".

Many years ago, I read that our old friend, Newton, saw a simple pattern for producing these **coefficients** without having to use Pascal's triangle as follows:

Consider $(x + h)^5$

$$= x^5 + \frac{5}{1}x^4h^1 + \frac{5\times4}{1\times2}x^3h^2 + \frac{5\times4\times3}{1\times2\times3}x^2h^3 + \frac{5\times4\times3\times2}{1\times2\times3\times4}x^1h^4 + \frac{5\times4\times3\times2\times1}{1\times2\times3\times4\times5}h^5$$

$$= x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

(Notice the indices of the ***x***'s reduce by 1 and the ***h***'s increase by 1)

I hope you see the lovely pattern how the coefficients are formed.

Generally it is this.... I have written the coefficients in **RED**

$$(x + h)^n = x^n + \frac{n}{1}x^{n-1}h^1 + \frac{n(n-1)}{1\times2}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{1\times2\times3}x^{n-3}h^3 + \dots$$

Eg. $(x + h)^3 = x^3 + \frac{3}{1}x^2h^1 + \frac{3\times2}{1\times2}x^1h^2 + \frac{3\times2\times1}{1\times2\times3}h^3$

$= x^3 + 3x^2h + 3xh^2 + h^3$

AND $(x + h)^4 = x^4 + \frac{4}{1}x^3h + \frac{4\times3}{1\times2}x^2h^2 + \frac{4\times3\times2}{1\times2\times3}xh^3 + \frac{4\times3\times2\times1}{1\times2\times3\times4}h^4$

$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

AND $(x + h)^5 = x^5 + \frac{5}{1}x^4h + \frac{5\times4}{1\times2}x^3h^2 + \frac{5\times4\times3}{1\times2\times3}x^2h^3 + \frac{5\times4\times3\times2\ x}{1\times2\times3\times4}h^4 + \frac{5\times4\times3\times2\times1}{1\times2\times3\times4\times5}h^5$

$= x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

$$(x + h)^{50} = x^{50} + \frac{50}{1}x^{49}h^1 + \frac{50\times49}{1\times2}x^{49}h^2 + \frac{50\times49\times48}{1\times2\times3}x^{48}h^3 + \ldots$$

EDIT:

The general form which I wrote earlier…

$(x + h)^n = x^n + \frac{n}{1}x^{n-1}h^1 + \frac{n(n-1)}{1\times2}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{1\times2\times3}x^{n-3}h^3 + \ldots$

…even can be applied when **n** is **negative** or a **fraction**!

eg $(1 - x)^{-1} = 1^{-1} + \frac{{}^{-}1}{1}(-x)^1 + \frac{{}^{-}1\times{}^{-}2}{1\times2}(-x)^2 + \frac{{}^{-}1\times{}^{-}2\times{}^{-}3}{1\times2\times3}(-x)^2 \ldots$

$= 1 + x + x^2 + x^3 \ldots$

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eg $(1 + x)^{½} = 1^{\frac{1}{2}} + \frac{\frac{1}{2}}{1}x^1 + \frac{\frac{1}{2}\times-\frac{1}{2}}{1\times2}x^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{1\times2\times3}x^3 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}\times-\frac{5}{2}}{1\times2\times3\times4}x^4$

$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \ldots$