HOW CAN sin(x) = 2.

The basic graph of **y = sin(x)** just has **real x** values and **real y** values.

The key to understanding my answer is that we can find some **complex values of x which still produce real y values!**

In order to allow **complex x values**, I need to write the x values as complex numbers so **I will replace x with x + iz**

Instead of just **y = sin(x),** I will use **y = sin(x + iz)**

Obviously I will need a **real x axis** and an **imaginary x axis** which I called **z.**

let y = sin(x + iz)
= sin(x) cos(iz) + cos(x) sin(iz)
= sin(x) cosh(z) + icos(x) sinh(z)

I know this looks awful but notice that certain values of x will ensure that we the **y values stay real.**

If x = $\frac{\pi}{2}$

Then **y** = $\sin\left(\frac{\pi}{2}\right) \times \cosh(z) + i\cos\left(\frac{\pi}{2}\right) \times \sinh(z)$

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so that **y** = **1** × **cosh(z)** + i × **0** = **cosh(z)**

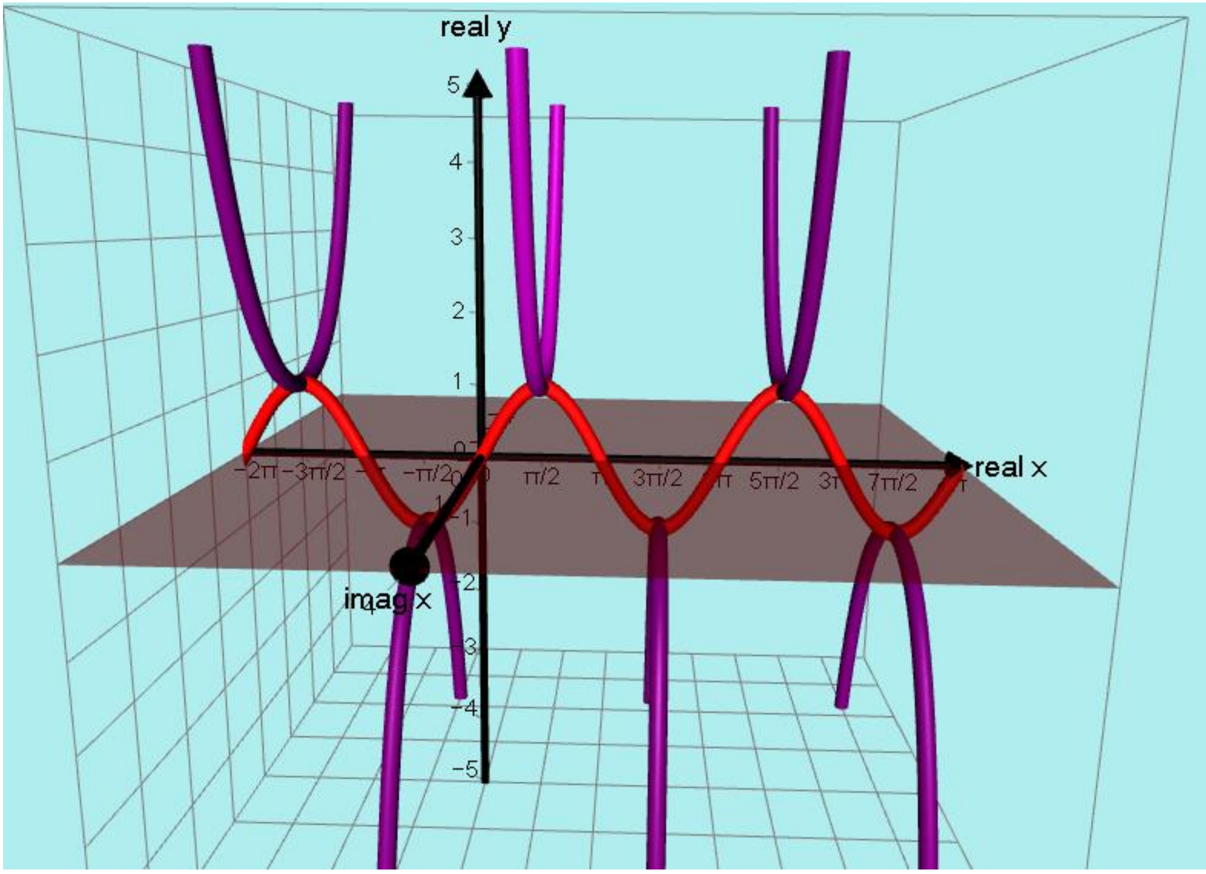
In fact for all values of **x** = $\frac{\pi}{2} + 2n\pi$ then **y = cosh(z)**

Also if x = $\frac{3\pi}{2}$

Then **y = –1×cosh(z) + i×0 = – cosh(z)**

In fact for all values of **x** = $\frac{3\pi}{2} + 2n\pi$ **then y = – cosh(z)**

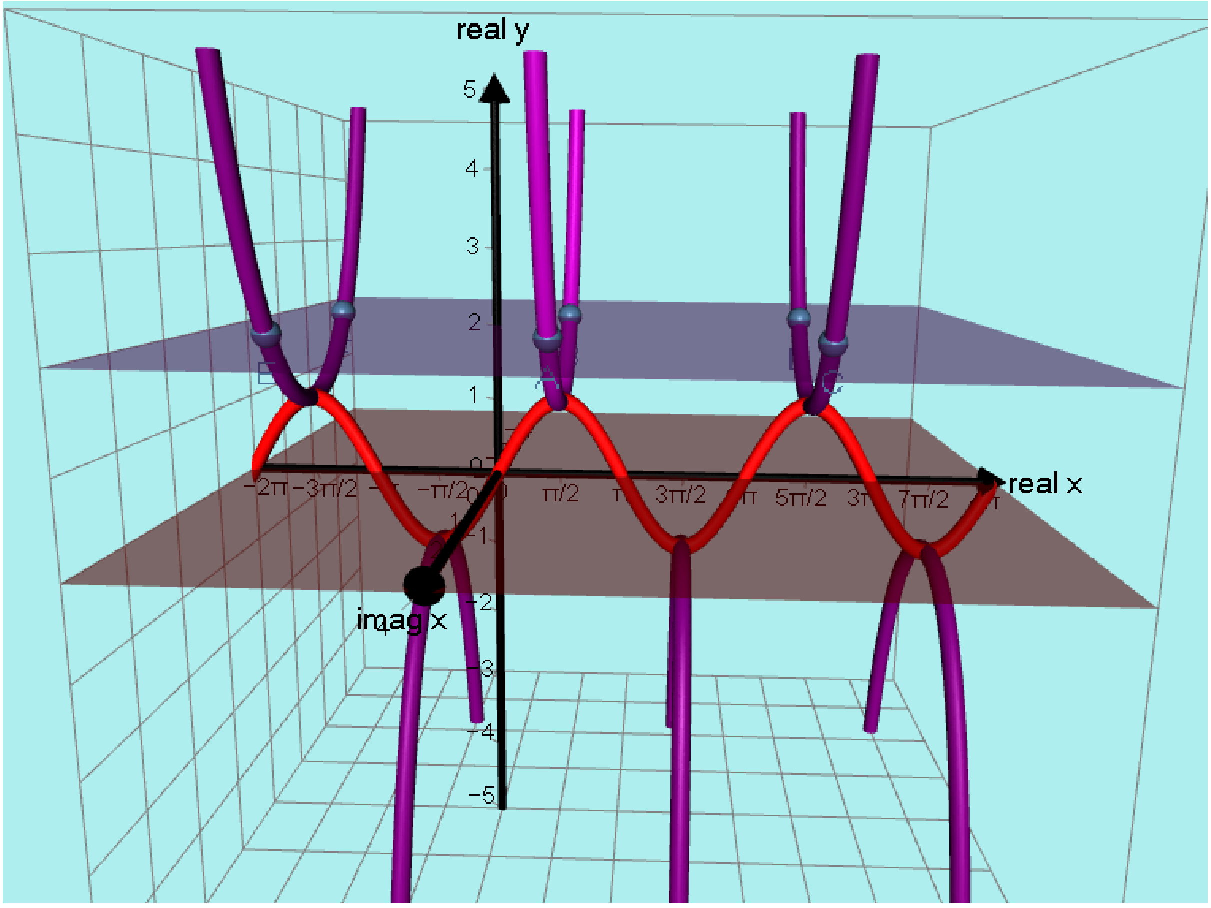
The graph of y = sin(x + iz) for REAL y values is:



This means that y = sin(x) is not restricted to y values between – 1 and + 1

Now consider where **sin(x + iz) = 2**

I will draw the **plane y = 2** and we see the intersection points:



The solutions where **sin(x +** ***iz*****) = 2** are where **cosh(z) = 2**
That is when **z = ±1.317*i***

From the diagram above, the values of **x +** ***iz*** which make **sin(x +** ***iz*****) = 2** are:

x + *i*z	Because sin(x + *i*z) = 2
$\frac{\pi}{2} \pm 1.317i$	$\sin\left(\frac{\pi}{2} \pm 1.317i\right) = 2$
$\frac{5\pi}{2} \pm 1.317i$	$\sin\left(\frac{5\pi}{2} \pm 1.317i\right) = 2$
$\frac{-3\pi}{2} \pm 1.317i$	$\sin\left(-\frac{3\pi}{2} \pm 1.317i\right) = 2$