Why can't we differentiate a power of x to get $\frac{1}{x}$?

A good way to think of this is from the aspect of differentiating simple powers of x.
When we do this there is one particular power missing!

Consider these carefully:

$y = x^4$ **so** $y' = 4x^3$

$y = x^3$ **so** $y' = 3x^2$

$y = x^2$ **so** $y' = 2x^1$

$y = x^1$ **so** $y' = 1x^0 = 1$

$y = x^{-1}$ **so** $y' = -1\,x^{-2} = \frac{-1}{x^2}$

$y = x^{-2}$ **so** $y' = -2\,x^{-3} = \frac{-2}{x^3}$

$y = x^{-3}$ **so** $y' = -3\,x^{-4} = \frac{-3}{x^4}$

Did you notice that we never get an answer of $\frac{1}{x}$ **?**

No matter what power of x we differentiate, we can never get the answer of $\frac{1}{x}$

so if we antidifferentiate $\frac{1}{x}$ it cannot become a power of x.

Of course, we find out later that the antiderivative of $\frac{1}{x}$ **is** $\ln(x)$