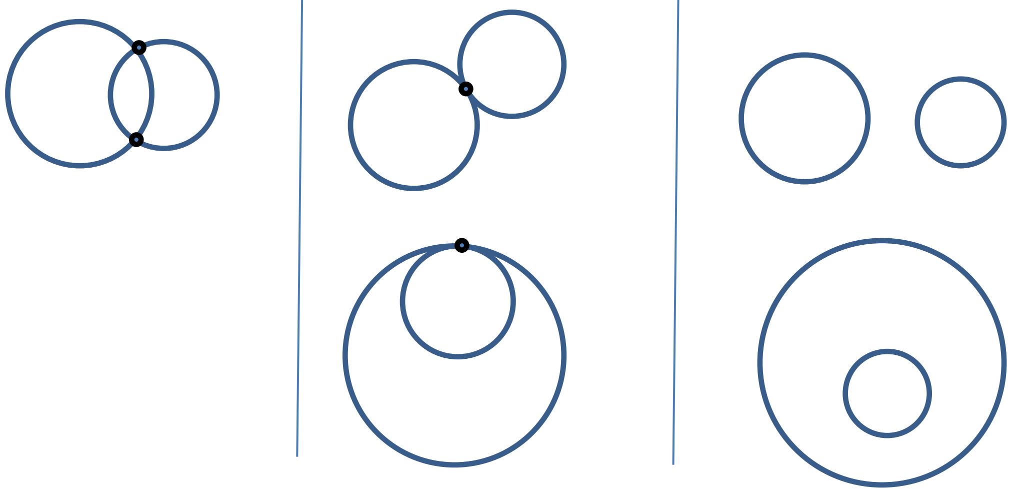
INTERSECTING CIRCLES.

I was recently telling someone how circles can intersect when I came across an astonishing conclusion!

I showed the obvious cases as follows:

2 intersections 1 intersection 0 intersections



For 2 intersections:

I showed how we obtain a **quadratic equation with 2 solutions.**

Example: $x^2 + y^2 = 5$ ----------------EquA

$and\ (x-2)^2 + (y-2)^2 = 1$ ------EquB

Expanding EquB: $x^2 - 4x + 4 + y^2 - 4y + 4 = 1$ ------EquC

Subtracting EquA: $-4x + 4 - 4y + 4 = -4$

$$y = 3 - x$$

Substitute in EquA: $x^2 + (3-x)^2 = 5$

$$2x^2 - 6x + 9 = 5$$

$$2x^2 - 6x + 4 = 0$$

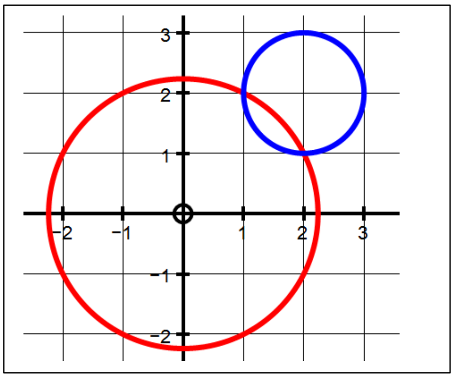
$$\mathbf{x^2 - 3x + 2 = 0}$$

$$\mathbf{(x-1)(x-2) = 0}$$

$$x = 1\ and\ x = 2$$

$$y = 2\ and\ y = 1$$

$$Intersection\ points\ (1,2)\ and\ (2,1)$$



For 1 intersection:

I showed how we obtain an **equation with 1 solution.**

Example:$y^2 = 4 - x^2$ ----------------EquA

$and\ (x-3)^2 + y^2 = 1$ --------------EquB

Substitute EquA into EquB:

$(x-3)^2 + (4 - x^2) = 1$

Expanding:

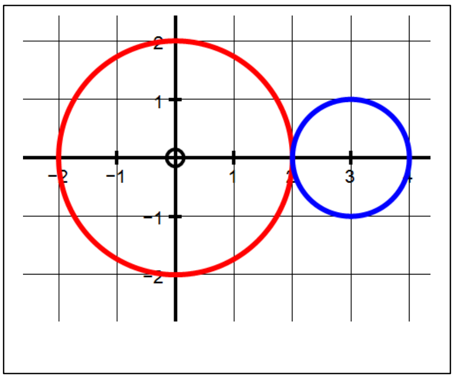
$x^2 - 6x + 9 + 4 - x^2 = 1$

$-6x + 12 = 0$

$x = 2$

$y = 0$

$Intersection\ point\ (2, 0)$



For 0 intersections:

I showed how we obtain an **equation with NO REAL solutions.**

Example: $x^2 = 16 - y^2$ ----------EquA

$and\ \ x^2 + (y-1)^2 = 1$ ----------EquB

Expanding EquB:

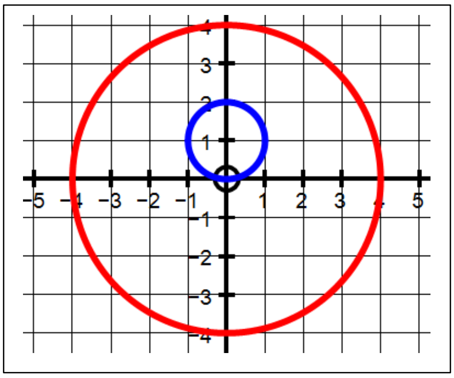
$x^2 + y^2 - 2y + 1 = 1$ ------EquC

Substituting EquA into EquB:

$16 - y^2 + y^2 - 2y + 1 = 1$

$2y = 16$

$y = 8$



This was quite a dilemma at first.

I was trying to show a quadratic with no real solutions such as:

$x^2 + x + 4 = 0$

However, I substituted ***y*** = 8 into EquA to obtain $x^2 = 16 - 64 = -48$

So if $x = \pm\sqrt{-48}$ there are no real intersections!

BUT then I started thinking….

"WHERE ARE THE IMAGINARY INTERSECTIONS?"

$If\ y = 8\ and\ x^2 = -48\ then\ x = \pm\sqrt{-48} = \pm 4i\sqrt{3}$

Somehow, these circle equations have intersection points at $(\pm 4i\sqrt{3},\ 8)$

Thinking back to my ***Phantom Graph*** theory, I found that a circle has an associated hyperbola attached! I will explain in full.

I will just consider **REAL y values** but I will allow **imaginary x values** (indicated by the answer I just obtained).
I will replace the REAL variable $\boldsymbol{x}$ by the complex version $\boldsymbol{x + iz}$
Equation A from the last section was $\boldsymbol{x^2 + y^2 = 16}$
This now becomes: $(\boldsymbol{x} + \boldsymbol{iz})^2 + y^2 = 16$

$$\boldsymbol{x^2 + 2xzi - z^2 + y^2 = 16}$$

$$\boldsymbol{\textit{so we get } y^2 = 16 - x^2 - 2xzi + z^2} \text{ --------------Equ C}$$

If y is to be REAL then the term $\boldsymbol{2xzi}$ **has to be zero.**
This means either *z = 0 or x = 0*

$\boldsymbol{If\ z = 0}$
Equ C becomes:

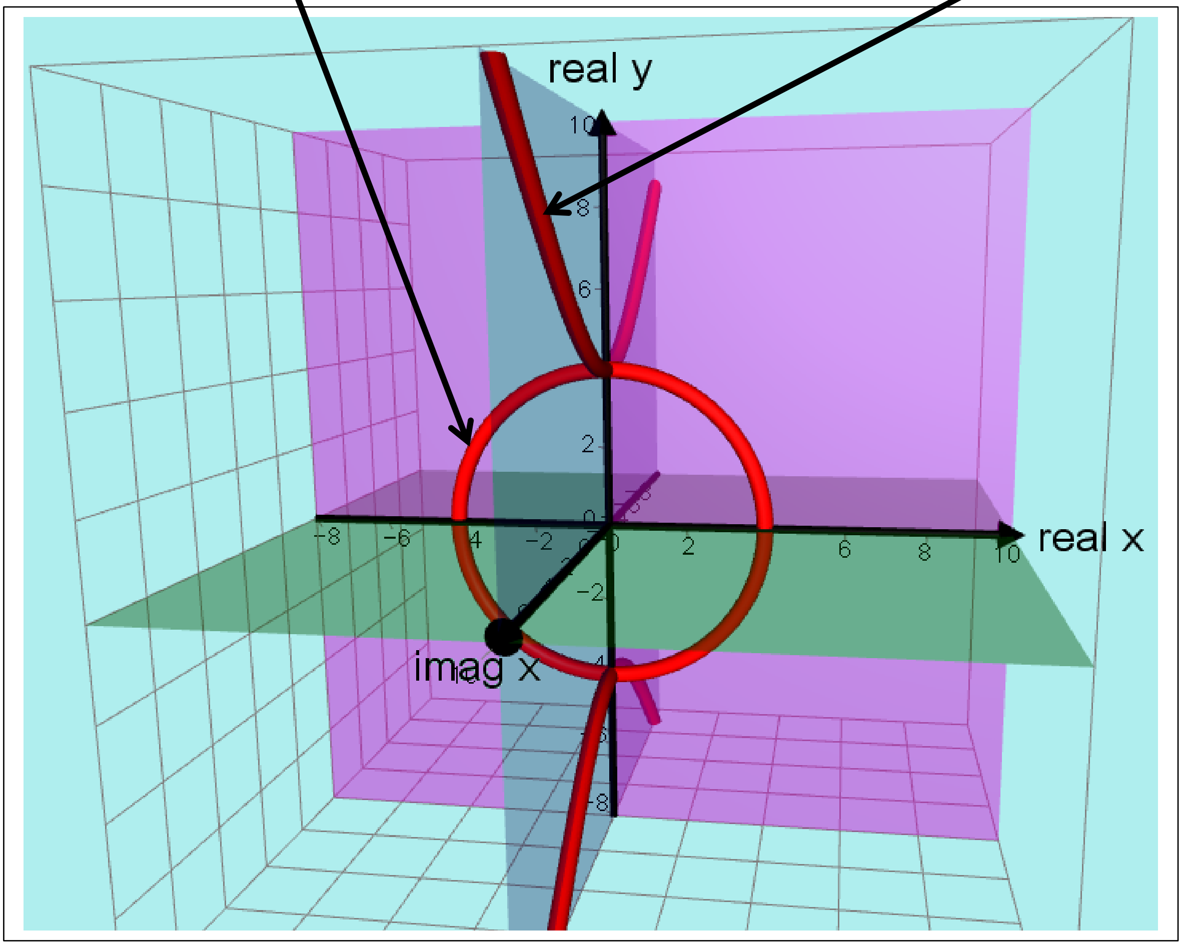
$$\boldsymbol{y^2 = 16 - x^2}$$

which is the original circle in the x, y plane.

$\boldsymbol{If\ x = 0}$
Equ C becomes:

$$\boldsymbol{y^2 = 16 + z^2}$$

which is the equation of the phantom hyperbola in the z, y plane



Equation B from the last section was $x^2 + (y-1)^2 = 1$

This now becomes: $(\boldsymbol{x} + \boldsymbol{iz})^2 + (y-1)^2 = 1$

$$\boldsymbol{x^2 + 2xzi - z^2} + (y-1)^2 = \mathbf{1}$$

$$\boldsymbol{so\ we\ get}\ (y-1)^2 = \mathbf{1} - \boldsymbol{x^2} - \boldsymbol{2xzi} + \boldsymbol{z^2}$$ **--------------Equ D**

If y is to be REAL then the term $\boldsymbol{2xzi}$ **has to be zero.**

This means either *z = 0 or x = 0*

$\boldsymbol{If\ z = 0}$

Equ D becomes:

$$(y-1)^2 = \mathbf{1} - \boldsymbol{x^2}$$

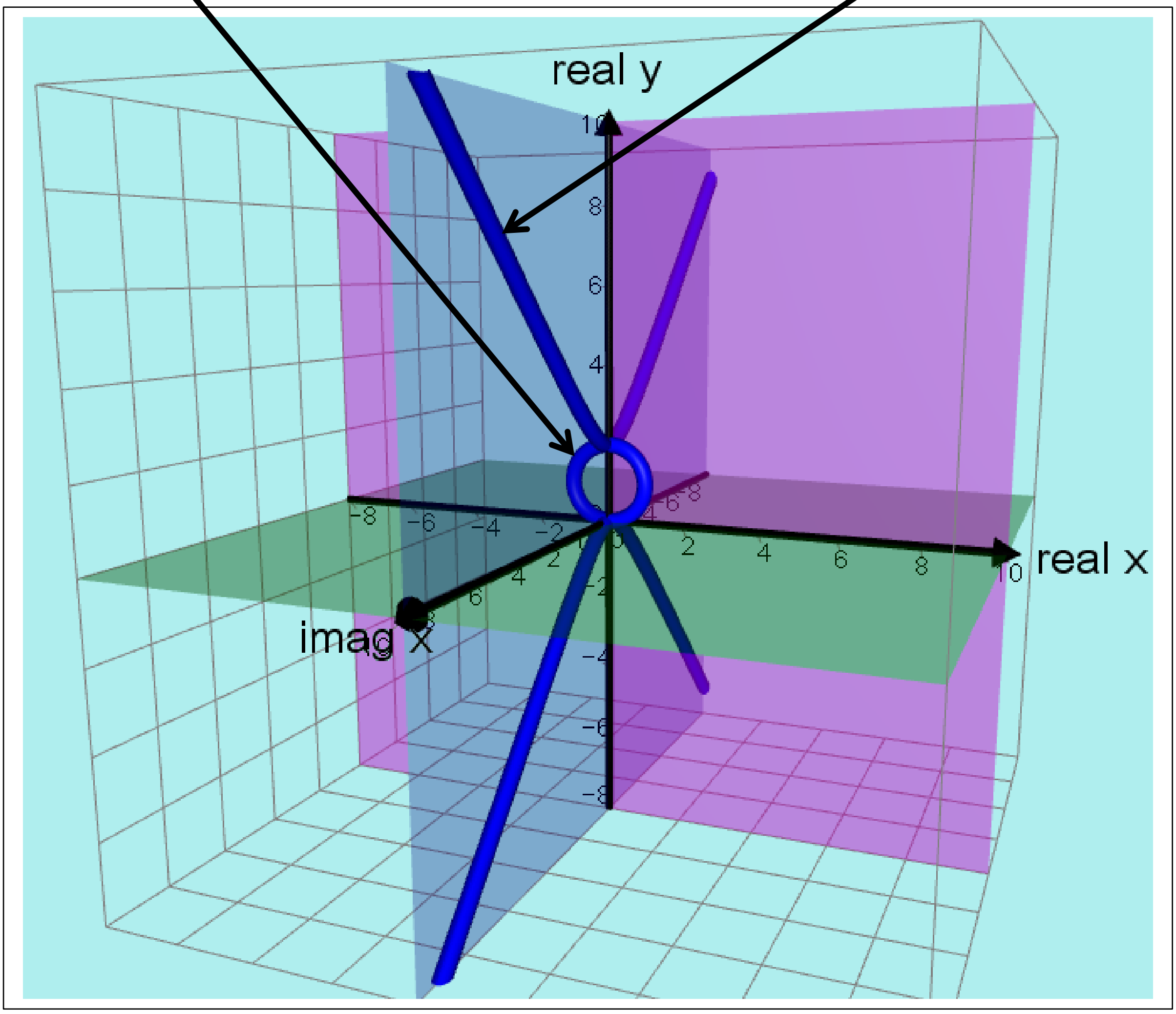
which is the original circle in the x, y plane.

$\boldsymbol{If\ x = 0}$

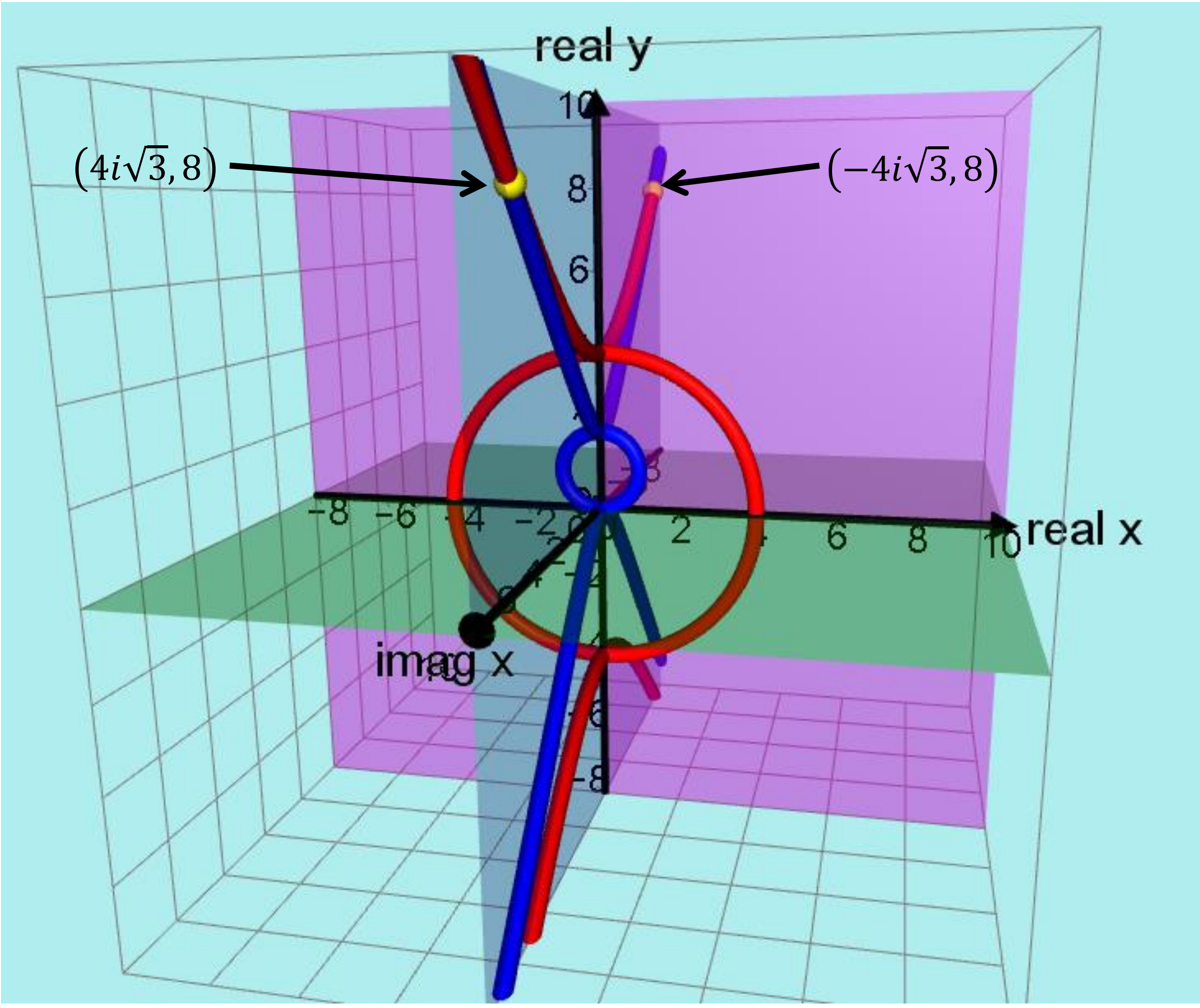
Equ D becomes:

$$(y-1)^2 = \mathbf{1} + \boldsymbol{z^2}$$

which is the equation of the phantom hyperbola in the z, y plane

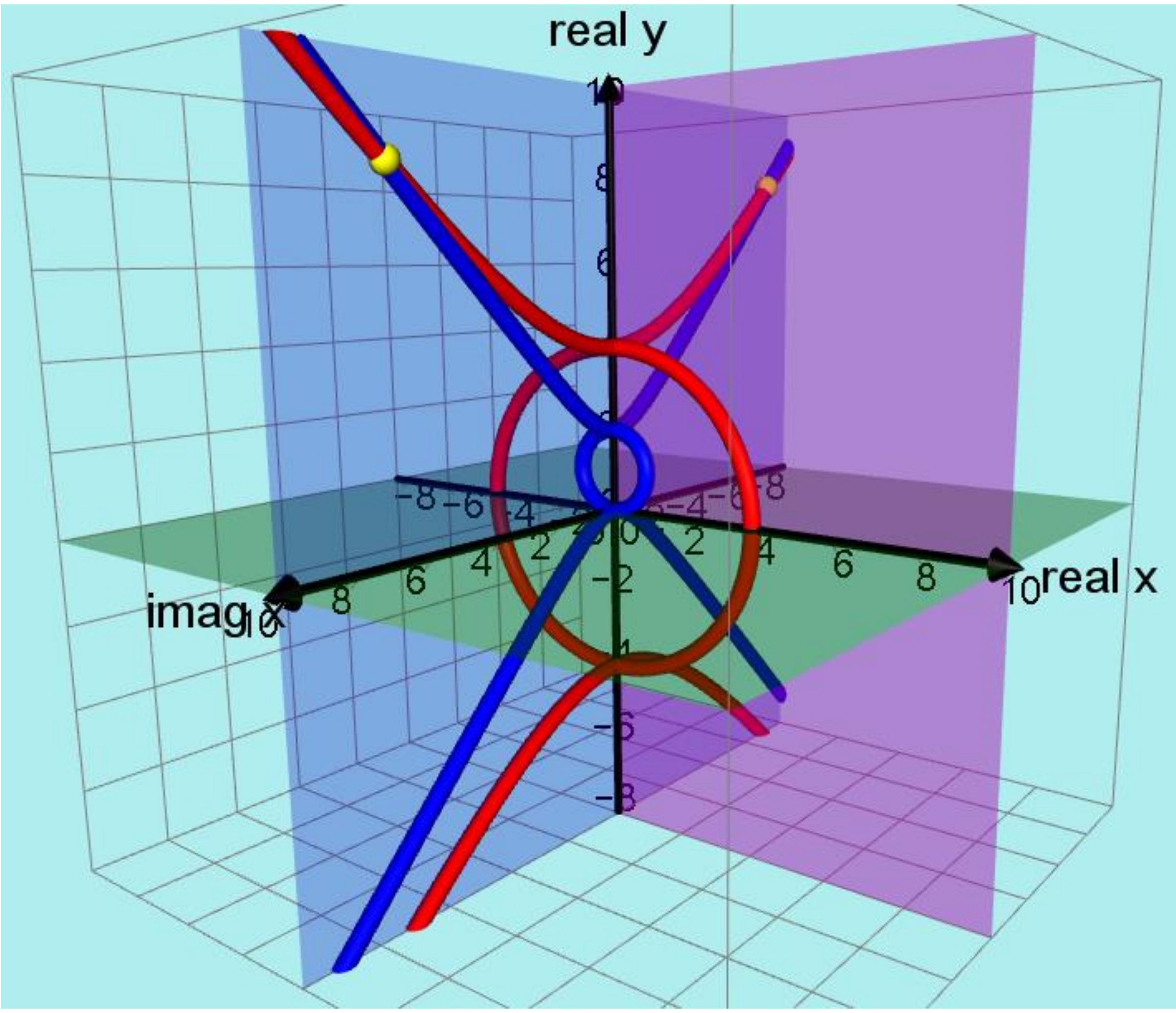


Finally we put these two graphs together and we can see that the phantom graphs DO intersect at the points $\left(\pm 4i\sqrt{3},\ 8\right)$ marked as yellow points.



The two lower sections of the phantom hyperbolae do not intersect.

Here is a different view of the graphs…



real y
real x
imag x