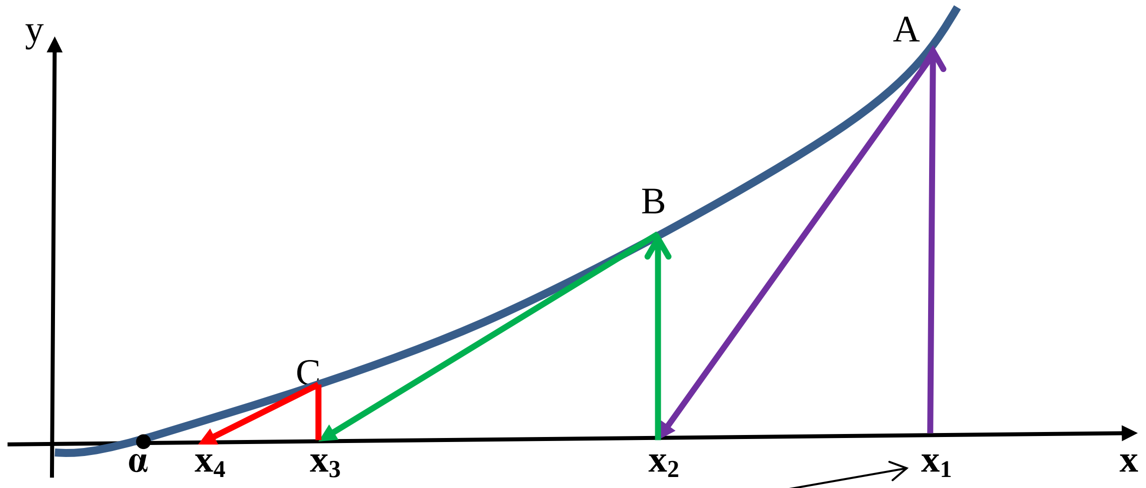
The Newton-Raphson Method for finding approximate solutions of equations.

Below is the graph of **y = f(x)** so the solution of **f(x) = 0** is the point where the graph crosses the **x** axis at **x = α.**

This diagram shows how the iterative process approaches the solution of the equation **f(x) = 0**.



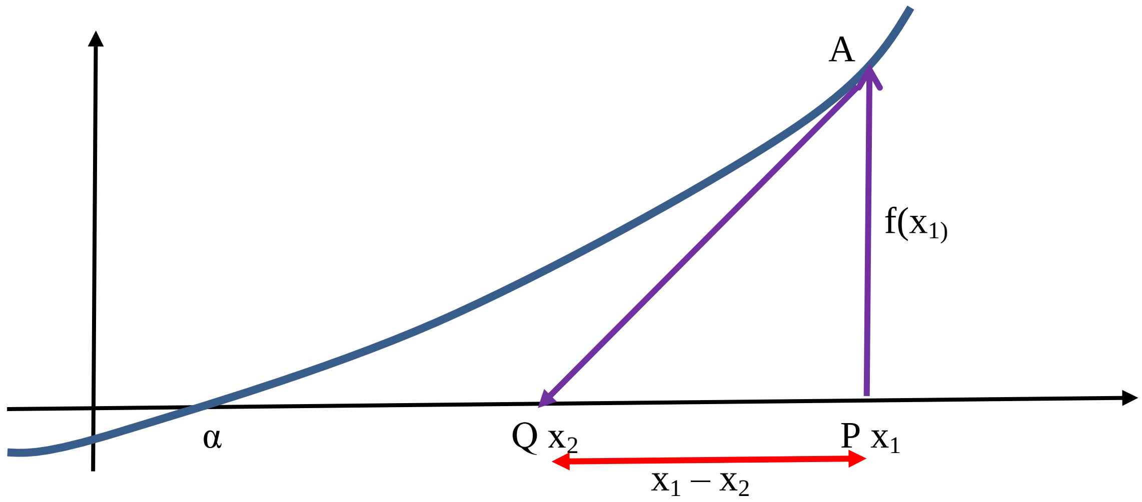
The 1st approximation is x_1.
Draw a vertical line from x_1 to the curve meeting it at point A.
A tangent is drawn from A meeting the x axis at the 2nd approximation x_2

Draw a vertical line from x_2 to the curve meeting it at point B.
A tangent is drawn from B meeting the x axis at the 3rd approximation x_3

Draw a vertical line from x_3 to the curve meeting it at point C.
A tangent is drawn from C meeting the x axis at the 4th approximation x_4

This process is continued until the approximation is deemed to be close enough to the solution **x = α**

We can make a simple formula to do this process as follows.



The 1st approximation is $\mathbf{x_1}$ so the distance **PA = f($\mathbf{x_1}$)**
The tangent at A is drawn which meets the x axis at Q where $\mathbf{x = x_2}$
The distance **PQ is $\mathbf{x_1 - x_2}$**
The gradient of the **tangent AQ** is $\dfrac{\mathbf{AP}}{\mathbf{PQ}} = \dfrac{\mathbf{f(x_1)}}{\mathbf{x_1 - x_2}}$

Another expression for the gradient of the **tangent AQ** is the derivative of the curve $\mathbf{y = f(x)}$ **at** $\mathbf{x = x_1}$. This is $\mathbf{f'(x_1)}$

Equating these two expressions we get:

$$\frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

so $$\frac{f(x_1)}{f'(x_1)} = x_1 - x_2$$

and so $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

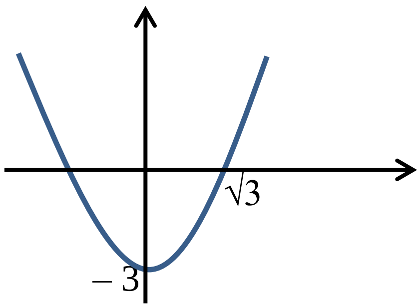
This is the iterative equation which we keep using:

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$	$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$	$x_5 = x_4 - \dfrac{f(x_4)}{f'(x_4)}$

In general we say: $x_{(n+1)} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Example: This method is very good for finding square roots.

Suppose we want $\sqrt{3}$
We make an equation $x = \sqrt{3}$ so that $x^2 = 3$
Now we consider the graph $y = x^2 - 3$
so that the graph will cross the x axis at $\sqrt{3}$



Now we use: $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

It is sometimes a good idea to simplify this formula for the specific case but it is not compulsory.

$$x_2 = x - \frac{(x^2 - 3)}{2x} = \frac{2x^2 - (x^2 - 3)}{2x} = \frac{x^2 + 3}{2x}$$

let us choose the 1st approximation $x_1 = 2$

so $x_2 = \dfrac{4 + 3}{4} = 1.75$

$x_1 = 2$	$\dfrac{4+3}{4}$	1.75
$x_2 = 1.75$	$\dfrac{1.75^2 + 3}{2 \times 1.75}$	1.732142857
$x_3 = 1.732142857$	$\dfrac{1.732142857^2 + 3}{2 \times 1.732142857}$	1.73205081
$x_4 = 1.73205081$	$\dfrac{1.73205081^2 + 3}{2 \times 1.73205081}$	1.732050808

It is quite clear that this converges very quickly to at least 9 significant figures already!

Generally, to find √N just use $\mathbf{x_{n+1} = \dfrac{x_n^2 + N}{2x_n}}$