What is the algebraic solution to the following system: the unit circle x^2+y^2=1 and y=x^2 ? When I graph these, it is obvious that there are 2 solutions.

If $y = x^2$ and $x^2 + y^2 = 1$

then $x^2 + x^4 = 1$

so $x^4 + x^2 - 1 = 0$

I would treat this as a QUADRATIC equation by letting $x^2 = z$ and so $x^4 = z^2$

The equation now looks like this…

$$z^2 + z - 1 = 0$$

Using the quadratic formula $z = \frac{-1 \pm \sqrt{(1+4)}}{2}$

$$= \frac{-1 \pm \sqrt{(5)}}{2}$$

$$\approx 0.618 \text{ or } -1.618$$

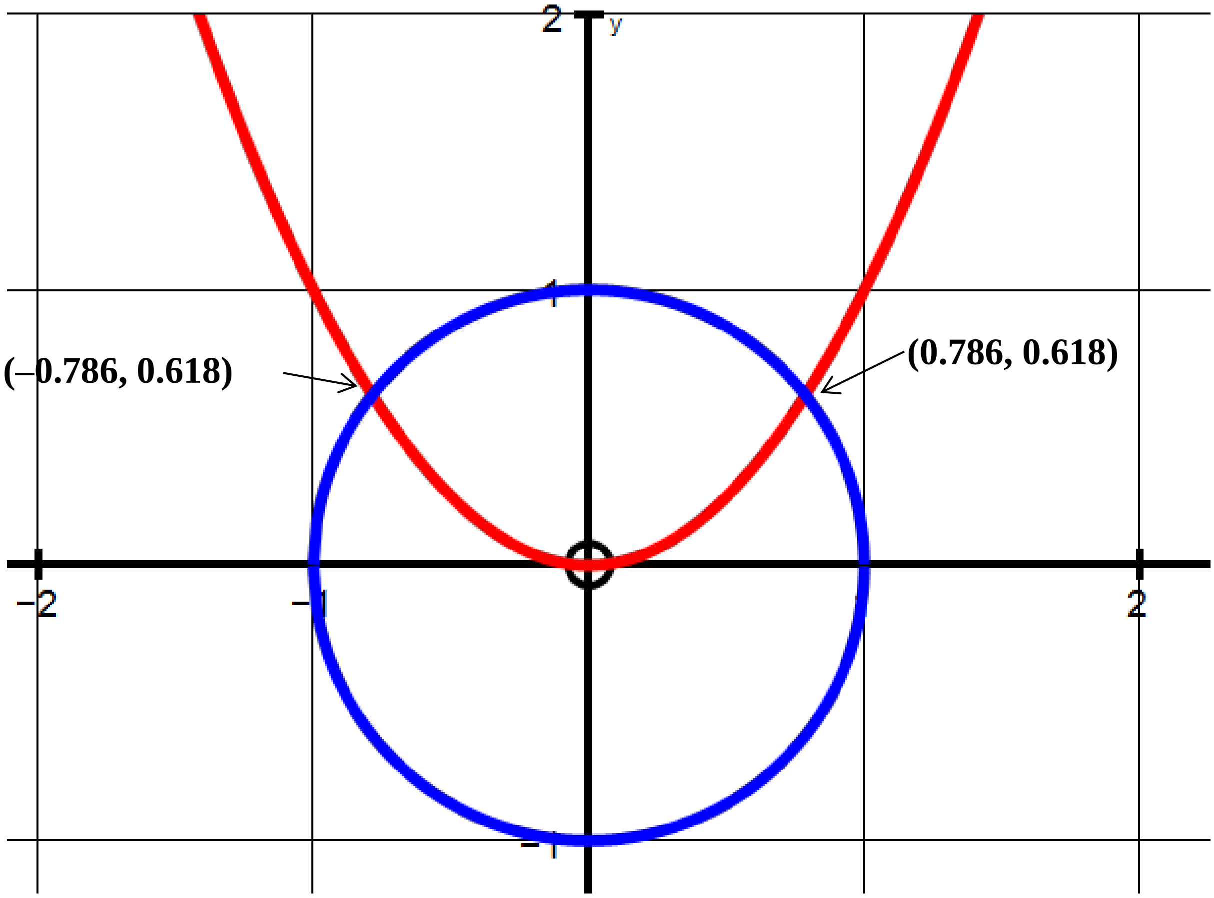
Now I have to find x

$x^2 = 0.618$ so $x = \pm\sqrt{0.618} = 0.786$ or -0.786

but if $x^2 = -1.618$ there are only imaginary solutions which we can't put on a 2D graph.

The two intersection points are (0.786, 0.618) and (– 0.786, 0.618)

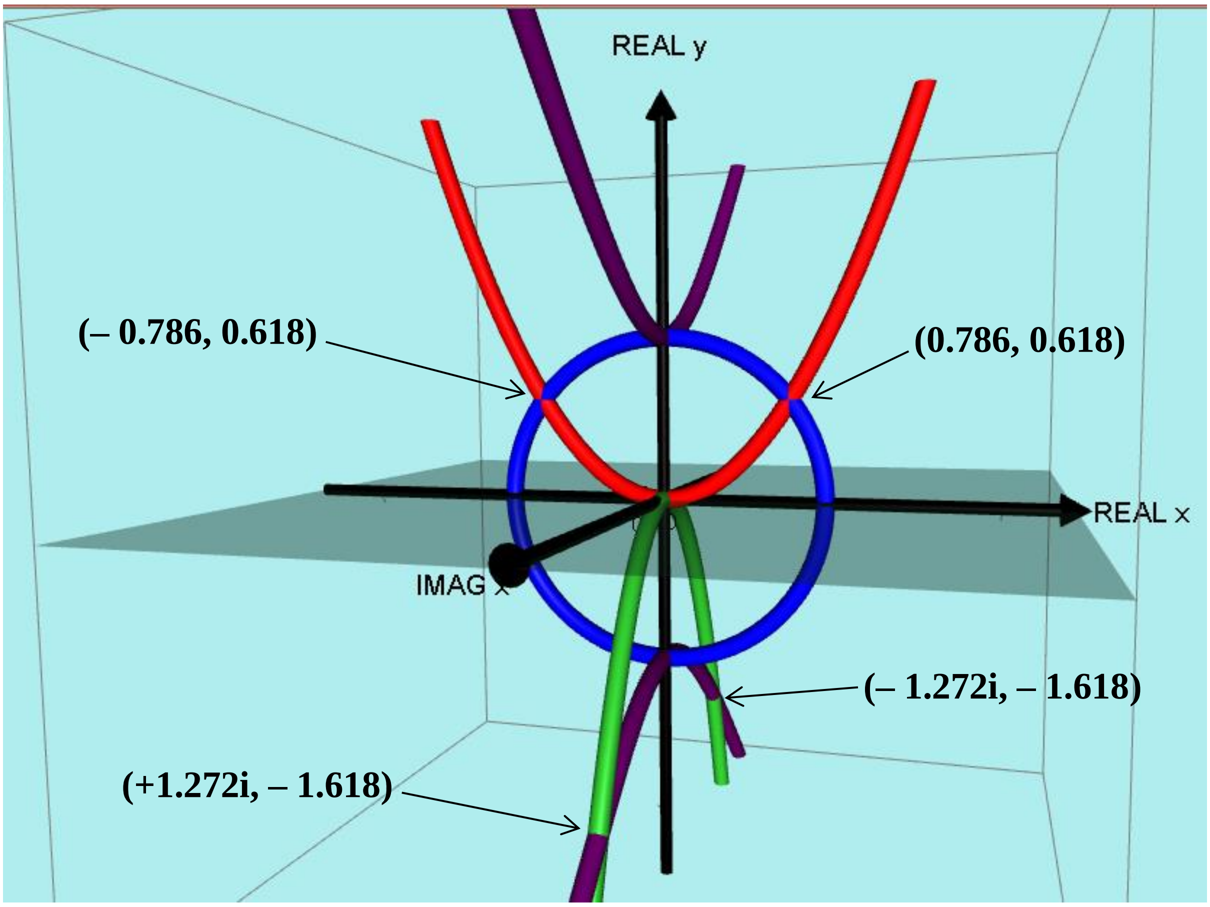
I love diagrams and graphs to enhance my answers!



NOW I have an unexpected treat! I can show you where the imaginary solutions are too!!!

There are extra imaginary points which satisfy $x^2 + y^2 = 1$ and $y = x^2$

Earlier I said $x^2 = -1.618$ so $x = \pm\sqrt{(-1.618)} = \mathbf{\pm 1.272i}$
and $y = -1.618$



I can’t fully explain this here but please see my website to find out more about these extra bits on graphs!

www.phantomgraphs.weebly.com