Why can’t we fold a piece of paper more than eight times?

Philip Lloyd, former Specialist Calculus Teacher and Mentor.

This question is an excellent application for the topic of INDICES at the level of 13- or 14-year-old students.

I wrote this lesson plan for other teachers to use.

It has been VERY popular!

I usually do it in two parts as follows…

PART 1

Ask the class “HOW THICK IS A PIECE OF PAPER?”

They may come up with ideas such as:

Sir, you can’t measure 1 piece with a ruler!

Or perhaps use a micrometer

Or measure 100 or 200 sheets and divide.

Get pupils to measure books or note pads or perhaps a phone book and come up with some sort of approximation.

(Hint: If the last page in a book is 400, it only has 200 pages!)

PART 2

CHALLENGE!

Offer a $10 prize for anyone who can fold an ordinary piece of writing paper IN HALF 10 times. (YOUR MONEY IS VERY SAFE!)

Let them try.

Some people **may** get to seven times.

Some may say they could do it if it were longer… so if possible, get a long roll of paper about 10 metres long and try.

You still will not get past about seven times!

The point of all this is to use INDICES to work out the thickness.

We produce a table as follows…

N^0 of folds	N^0 of sheets .thick	Power of 2
0	1	2^0
1	2	2^1
2	4	2^2
3	8	2^3
4	16	2^4
5	32	2^5
6	64	2^6
7	128	2^7
8	256	2^8
9	512	2^9
10	1024	2^{10}

Notes: FOR TEACHERS

1. If we could fold a piece 10 times, it would be over 1000 sheets thick!!!
(Similar to a telephone book.)

2. Point out clearly that the thicknesses are powers of 2 and after 6 folds thickness = 2^6

3. Notice that after zero folds it is of thickness 1 which shows that $2^0 = 1$

It is really worthwhile it to try this for yourself with a long piece of paper.

It gets so incredibly thick and so really hard to fold quite quickly!

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SPECIAL EDIT!

Somebody asked me “what is the difference between FOLDING the paper in half and WINDING or ROLLING the paper around”.

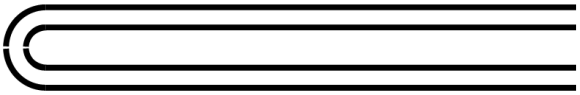
I spent hours producing these diagrams to explain this…

“FOLDING” A SHEET OF PAPER IN HALF AS MANY TIMES AS POSSIBLE.

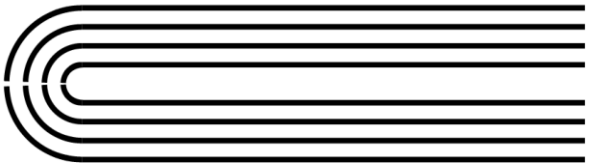
Single sheet of paper (not yet folded in half)

1 fold (2 sheets thick) = 2^1

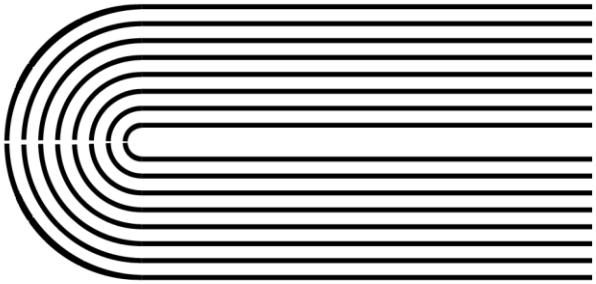
2 folds (4 sheets thick) = 2^2



3 folds (8 sheets thick) = 2^3



4 folds (16 sheets thick) = 2^4

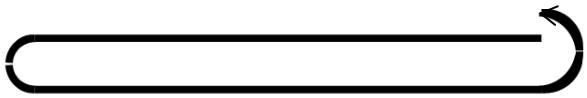


5 folds (32 sheets thick) = 2^5
6 folds (64 sheets thick) = 2^6
7 folds (128 sheets thick) = 2^7
8 folds (256 sheets thick) = 2^8
9 folds (512 sheets thick) = 2^9
10 folds (1024 sheets thick) = 2^8

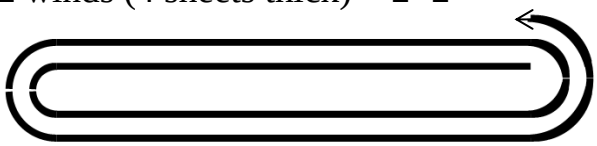
“WINDING” or “ROLLING” A SHEET OF PAPER.

Single sheet of paper not wound yet.

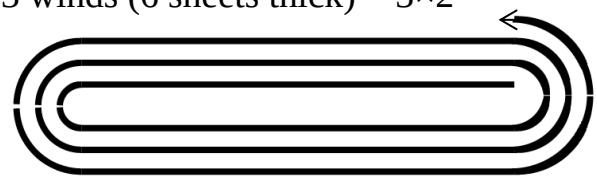
1 wind (2 sheets thick) = 1×2



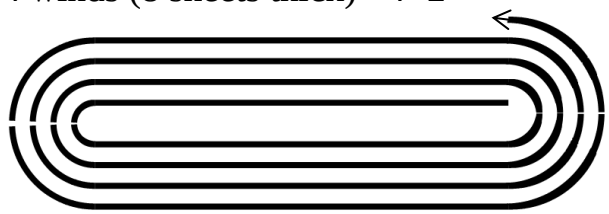
2 winds (4 sheets thick) = 2×2



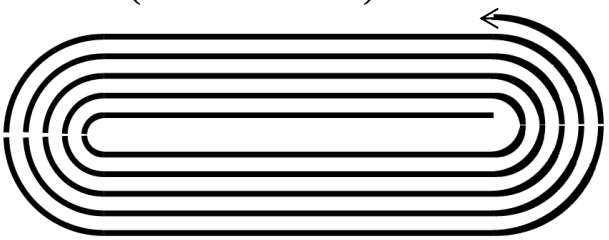
3 winds (6 sheets thick) = 3×2



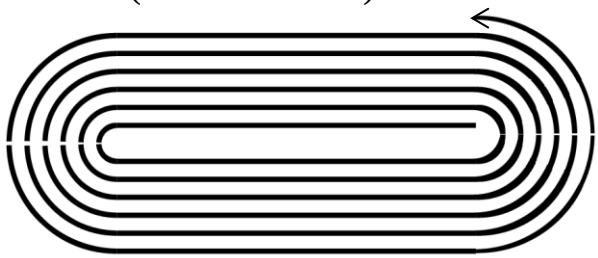
4 winds (8 sheets thick) =4×2



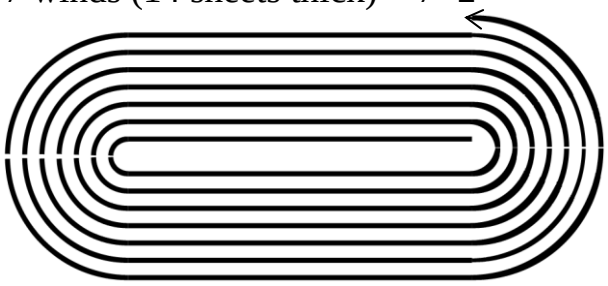
5 winds (10 sheets thick) =5×2



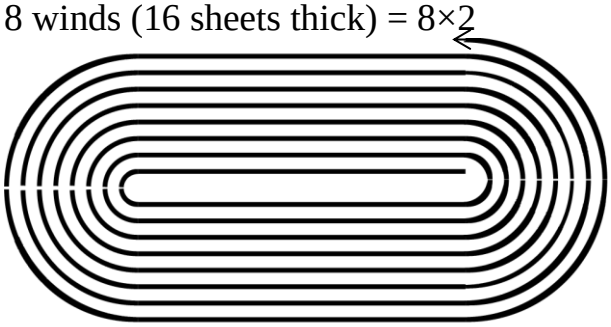
6 winds (12 sheets thick) = 6×2



7 winds (14 sheets thick) = 7×2



8 winds (16 sheets thick) = 8×2



Notice that 8 WINDS = 16 sheets thick but only 4 FOLDS = 16 sheets thick!!
Also 10 winds would be only 20 sheets thick but 10 folds would be 2^{10} = 1024 sheets thick!!!

The comparative graphs show this very well.
The maximum I can show on the graph is 7 folds = 128 sheets thick and 7 winds is only 14 sheets thick.