This is a simple classroom poster to keep these concepts clear for students.

$y = x(x - 3)^2$

$= x^3 - 6x^2 + 9x$

(cubic curve)

$y' = 3x^2 - 12x + 9$

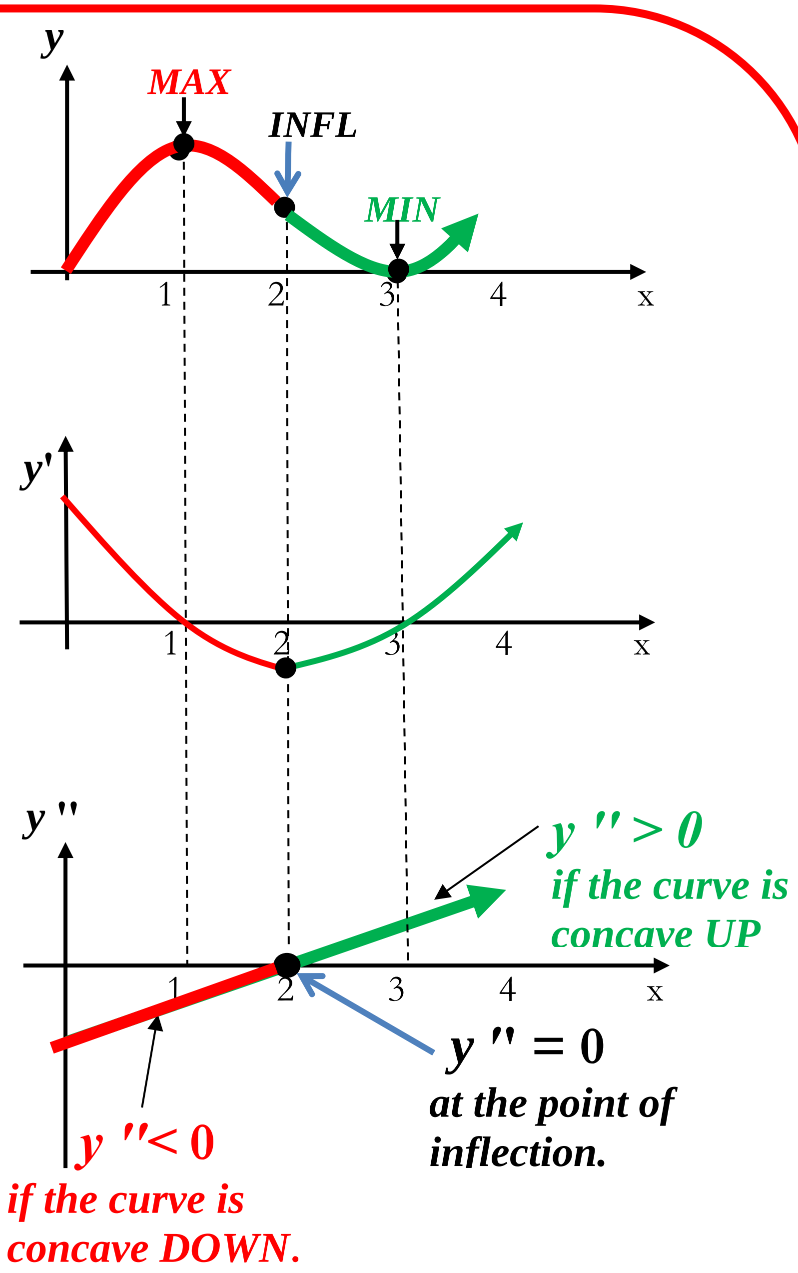
$= 3(x^2 - 4x + 3)$

$= 3(x - 1)(x - 3)$

(parabola)

$y'' = 6x - 12$

(line graph)



<u>**NOTICE THESE THREE POINTS:**</u>

When the cubic has a MAXIMUM the 2nd derivative is a NEGATIVE number.
When the cubic has a MINIMUM the 2nd derivative is a POSITIVE number.
When the cubic has an INFLECTION point the 2nd derivative is ZERO.

This was a Question from the QUORA website:

Why is it that when f''(x) = o this represents a point of inflection on the curve y=f(x)

NB Actually, the condition that $\frac{d^2y}{dx^2} = 0$ does not always mean that the curve will have an inflection point. I will cover this point later.

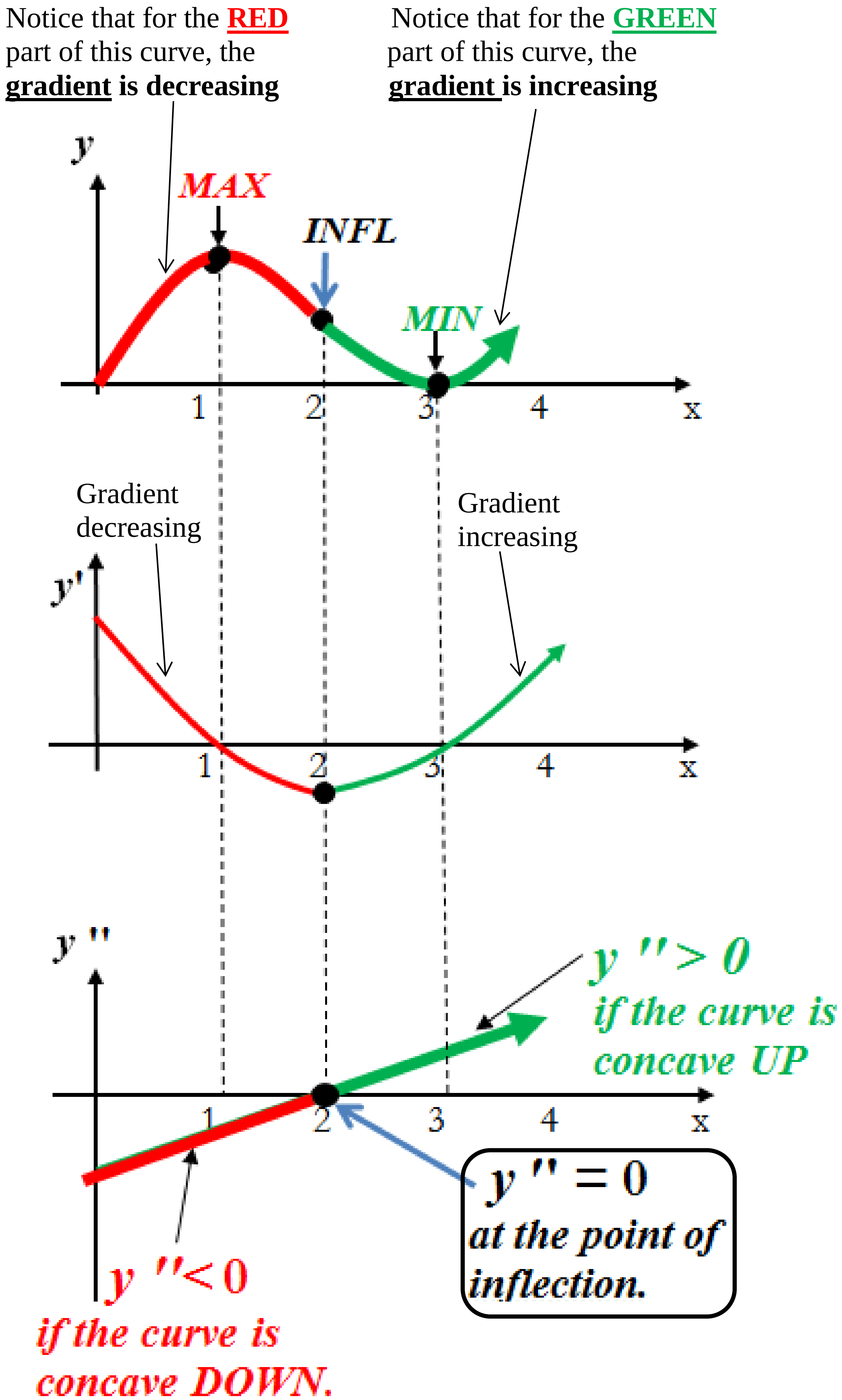
A point of inflection is:

“a point where the gradient stops **increasing** and starts **decreasing**”

OR the other way round:

“a point where the gradient stops **decreasing** and starts **increasing**”.

I will use the curve $\boldsymbol{y = x(x-3)^2}$ as an example…

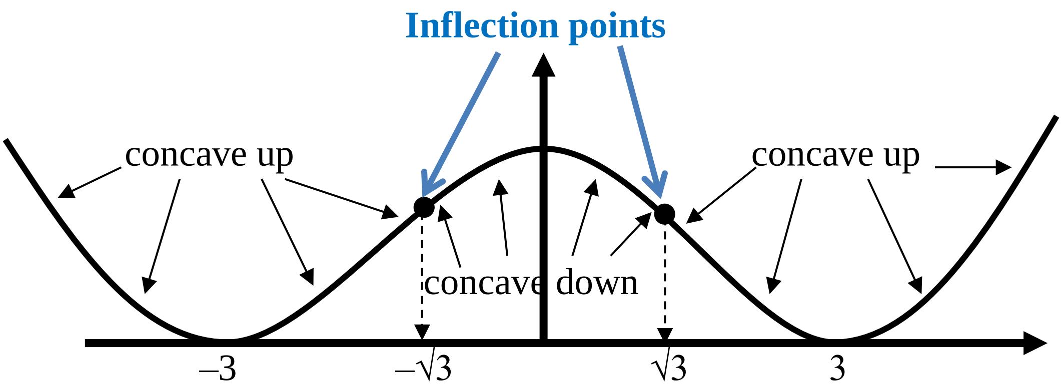

Notice that for the RED part of this curve, the gradient is decreasing
Notice that for the GREEN part of this curve, the gradient is increasing
y
MAX
INFL
MIN
1
2
3
4
x
Gradient decreasing
Gradient increasing
y'
1
2
3
4
x
y ''
y '' > 0
if the curve is concave UP
1
2
3
4
x
y '' = 0
at the point of inflection.
y ''< 0
if the curve is concave DOWN.

It should be noticed that **CONCAVITY** changes at inflection points.

Below I have drawn the curve:

$$y = (x + 3)^2(x - 3)^2$$

which has two inflection points at $\mathbf{x = \pm\sqrt{3}}$



Concave down if $\frac{d^2y}{dx^2} > 0$ *so* $12x^2 - 36 > 0$ *ie* $x^2 > 3$

The curve is CONCAVE DOWN for $-\sqrt{3} < x < \sqrt{3}$

You could just say that the curve is <u>concave down</u> if $-\sqrt{3} < x < \sqrt{3}$ **because there is a <u>Maximum point in between</u> or <u>if $y'' > 0$</u>.**

I have prepared some very short video demonstrations showing how the gradient changes from increasing to decreasing (or vice versa)

<u>POINTS OF INFLECTION SCREENCAST VIDEOS</u>

http://screencast.com/t/wnmfDn2Fcn
http://screencast.com/t/UHhMU9Gv
http://screencast.com/t/5BoYS0uN

I should also mention that $\frac{d^2y}{dx^2} = 0$ **does not guarantee** a point of inflection.

A good simple example is $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

$$\frac{d^2y}{dx^2} = 12x^2 = 0 \; if \; x = 0$$

But the curve has a minimum point not an inflection point!

Here is why!!!

Watch as the inflection points on the curve $y = (x^2 - a^2)^2$ slowly move together as the value of ***"a"*** approaches zero and the curve becomes $y = x^4$

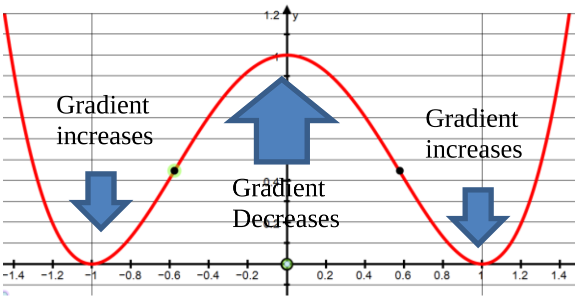
Vanishing points of inflection for $y = x^4$

https://www.screencast.com/t/46szQdm3yW

Vanishing points of inflection (advanced)

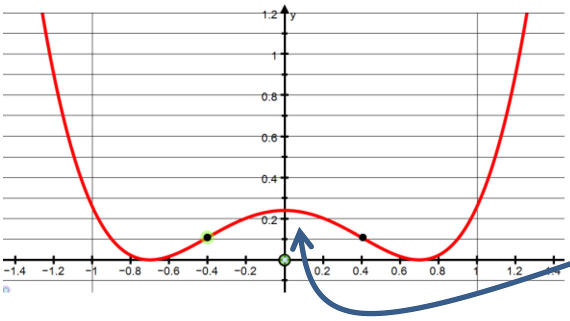
http://screencast.com/t/vK9HMkewE

See diagrams below…



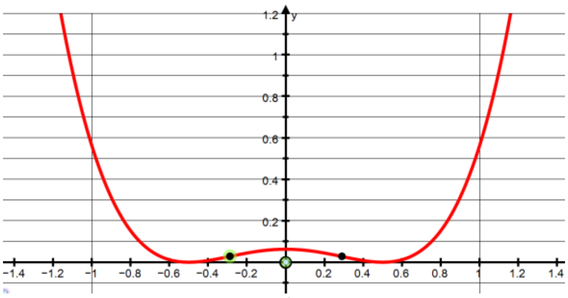
Here I drew the graph of
y = (x² – a²)² starting with ***a = 1***

I have worked out that the 2 inflection points are at $\left(\frac{\pm a}{\sqrt{3}}, \frac{4a^4}{9}\right)$

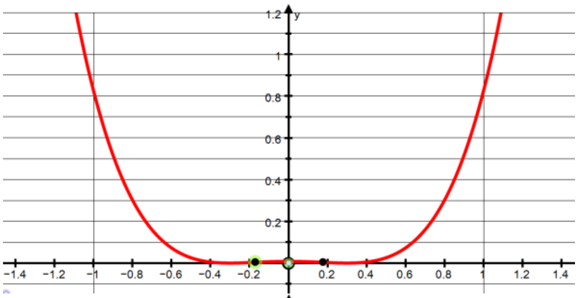


I then started to reduce the value of ***a*** and you can see the inflection points are moving closer to each other.

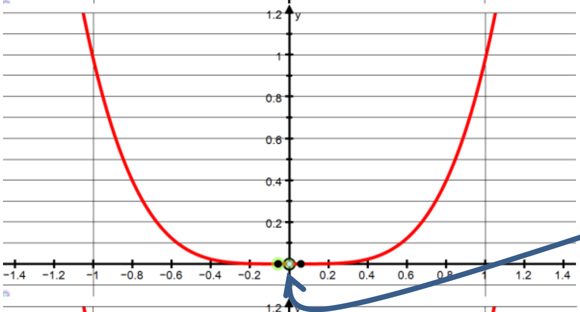
The curve is still concave down between the infection points



Here ***a = 0.5***

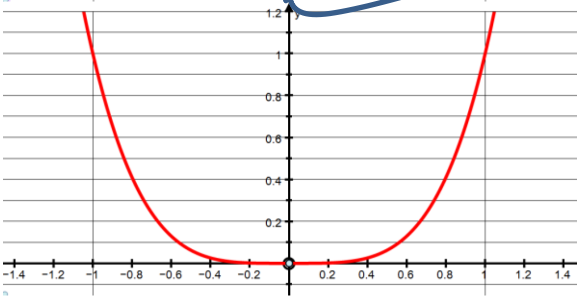


Here ***a = 0.3***



Here ***a = 0.1***

The curve is still concave down between these infection points!!!



Finally a = 0 and the two inflection points have coincided at (0, 0)
But actually they have vanished!
Because the curve is no longer concave down!
The curve has finally become *y = x⁴* which has no inflection points even though $\frac{d^2y}{dx^2} = 0$